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Optimal Scanning Protocol for Optical Tomography

Mahshad Javidan, *Student Member, IEEE*, Hadi Esfandi, and Ramin Pashaie, *Member, IEEE*

***Abstract*— Tomography is a two step process in which the sample under test is first scanned by the hardware of the system to acquire data and then the operating software reconstruct images from the gathered information. The main objective of this work is to optimize the scanning process to acquire maximum amount of information in each measurement when the system is scanning the sample. By exploiting our prior information about the sample and using estimation theory, we developed a systematic approach to implement the optimal scanning protocol. Results of this study provide strong evidence that the developed algorithms can speed up data acquisition. Also it is shown that the proposed method can reduce the impact of noise as well as improving the reconstruction error while performing less number of measurements.**

***Clinical relevance*— The proposed method can enhance data acquisition time, exposure dosage and cost of operation in medical applications of tomography.**

I. INTRODUCTION

Tomography is a type of imaging modality that captures the three-dimensional distribution of certain properties (e.g., contrast changes in absorption, fluorescence, or scattering parameters) in the sample. In this form of imaging, one or an array of sources radiate energy into the medium and an array of detectors on the output side records scattered field. In the past decades, researchers proposed methods to improve the efficiency of data acquisition process in tomography applications. One strategy is to construct a series of spatial illumination patterns that take advantage of the degree of freedom provided by external illumination. Various scanning protocols with specific illumination patterns have been proposed in the past [1]–[4]. However, such methods are usually only suitable for some applications and do not offer significant improvement. In a separate approach, researchers concentrate on the assessment of scanner's geometry or the design of illumination patterns to enhance the conditioning of the system matrix [5]–[8]. For instance, [9] obtained illumination patterns by improving the conditioning of the Fisher information matrix. Obviously, raster scanning is another popular scanning protocol used in many tomography systems. In this imaging technique, only one source is illuminating power during each measurement while all detectors are recording, thereby minimizing the linear dependency of the measurements [10].

The simplicity of the raster scanning algorithm and its capability in exploring the object with all possible independent measurements is an obvious benefit of the protocol. However, raster scanning is time consuming and not suitable for many imaging applications where the sample is dynamically evolving. To expedite data acquisition process, we need to reduce the number of measurements by making each measurement as informative as possible. One approach is to take advantage of the information content we already have about the object in the scanner before scanning. In this study, we apply some aspects of estimation theory to include prior information in the design of optimal illumination pattern. We propose a method based on the Kalman filter theory to compute optimal illumination patterns in data acquisition process and reconstruct images of acceptable quality from a smaller number of measurements.

II. OPTIMAL ILLUMINATION PATTERN DESIGN

Tomographic imaging problem consists of a forward problem and an inverse problem. The forward problem describes energy propagation in the medium from sources to detectors and the inverse problem uses the collected data to reconstruct the image. Prior to any measurement, we have an initial estimation of the distribution we intend to reveal. By using the forward model and this estimation, we can predict the outcome of any measurement. Each time we make an observation, the result can be used to update our estimation. The objective here is to create a mathematical framework that describes how we should update our estimation based on the forward model predictions and partial observations.

Here, we adopt diffusion model for the propagation of wave in tissue. The intensity of the scattered field at the location of the ith detector, $\bar{r}_{d_i}$ is

$$\phi_s(\bar{r}_{d_i}) \simeq \sum_{j=1}^{S}\sum_{k=1}^{V} I_j G_2(\bar{r}_{d_i};\bar{r}_{v_k}) G_1(\bar{r}_{v_k};\bar{r}_{s_j})\eta_k. \quad (1)$$

The variable I_j is the intensity of the field, illuminated by the jth source. $G(\bar{r};\bar{r}')$ is the Green's function which models the diffusion of the field for the selected scanner geometry. $\bar{r}_{v_k}$ is the vector that points to the center of the kth voxel which has the total scattering potential of η_k. In vector notation,

$$\bar{\phi}_s = \bar{\bar{W}}\bar{\eta} + \bar{\varepsilon}, \quad (2)$$

where $\bar{\phi}_s$ is a vector with D elements (D is the number of detectors), $\bar{\eta}$ is the estimation vector with V volume elements, and $\bar{\varepsilon} \sim \mathcal{N}(0, \bar{\bar{R}})$, is the measurement noise which

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The authors are with the computer and electrical engineering and computer science department, Florida Atlantic University, Boca Raton, FL, 33432 USA {mjavidan2021, hesfandi2021, rpashaie}@fau.edu

has Gaussian distribution. $\bar{\bar{W}}_{D\times V} = \bar{\bar{G}}_2 \cdot diag(\bar{\bar{G}}_1 \cdot \bar{I})$ is the weight matrix which depends on device specifics such as source-detector geometry and illumination pattern, $\bar{I}$.

Suppose that our prior estimate of the vector $\bar{\eta}$ is represented by $\hat{\eta}^{n|n-1}$ at the time of nth measurement. We use this estimate to predict the output of measurement which is $\bar{\bar{W}}^n\hat{\eta}^{n|n-1}$. After we make a measurement, $\bar{\phi}_s^n$, we need to combine the two pieces of information (model prediction and measurement output) in order to form a posterior estimate $\hat{\eta}^{n|n}$. This procedure is well aligned with the function of Kalman filter which combines theoretical predictions with noise contaminated partial observations to compute the updated minimum-variance estimation.

$$\hat{\eta}^{n|n} = \hat{\eta}^{n|n-1} + \bar{\bar{K}}^n \left[\bar{\phi}_s^n - \bar{\bar{W}}^n\hat{\eta}^{n|n-1}\right]. \quad (3)$$

Here, $\bar{\bar{K}}^n$ is the Kalman gain. Uncertainties in the prior and posterior estimations are modeled by covariance matrices: $\bar{\bar{P}}^{n|n-1} = Cov\left(\hat{\eta}^{n|n-1}\right)$ and $\bar{\bar{P}}^{n|n} = Cov\left(\hat{\eta}^{n|n}\right)$, respectively. Following the theory of Kalman filter, the uncertainty of the updated estimation (posterior covariance matrix) is given by:

$$\bar{\bar{P}}^{(n|n)} = \left[\bar{\bar{U}} - \bar{\bar{K}}^n\bar{\bar{W}}^n\right]\bar{\bar{P}}^{(n|n-1)}\left[\bar{\bar{U}} - \bar{\bar{K}}^n\bar{\bar{W}}^n\right]^T + \bar{\bar{K}}^n\bar{\bar{R}}(\bar{\bar{K}}^n)^T, \quad (4)$$

where $\bar{\bar{U}}$ is the identity matrix.

In this study, our aim is to find out which measurement leads to a posterior estimate $\hat{\eta}^{n|n}$ that is as certain as possible. This means that we need to minimize uncertainty of the posterior estimation. Since our uncertainty is modeled by a matrix, we use the trace of the covariance matrix as a measure of remaining uncertainty in the estimation. Trace is the norm that should be minimized by selecting the best measurement via designing the best illumination pattern. For simplicity, we first start by analyzing scanners with a single-detector and then we generalize the concept to scanners with multiple detectors.

A. Single-Detector Scanner

The Kalman gain is a column vector in the single-detector case, and the measurement noise is a scalar $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. Therefore, trace of the posterior covariance can be written as:

$$Tr[\bar{\bar{P}}^{n|n}] = Tr[\bar{\bar{P}}^{n|n-1}] - 2\bar{W}^n\bar{\bar{P}}^{n|n-1}\bar{K}^n + \left[\bar{W}^n\bar{\bar{P}}^{n|n-1}(\bar{W}^n)^T + \sigma^2\right](\bar{K}^n)^T\bar{K}^n. \quad (5)$$

To minimize this trace, we first assume that $\bar{W}^n$ is constant and we minimize $Tr(\bar{\bar{P}}^{n|n})$ with respect to Kalman gain. Solving $\nabla_{\bar{K}^n} Tr[\bar{\bar{P}}^{n|n}] = 0$ results in:

$$\bar{K}^{*n} = \left[\bar{W}^n\bar{\bar{P}}^{n|n-1}(\bar{W}^n)^T + \sigma^2\right]^{-1}\bar{\bar{P}}^{n|n-1}(\bar{W}^n)^T. \quad (6)$$

Now, given the optimal Kalman gain, we formulate the problem to search for the optimal illumination pattern, $\bar{I}^{(n)}$.

$$\begin{aligned} \bar{I}^{*n} = argmin_{\bar{I}^n} \quad & \frac{1}{2}(\bar{I}^n)^T\bar{\bar{A}}^{n|n-1}\bar{I}^n + \bar{b}^T\bar{I}^n, \\ s.t. \quad & I_j^{min} \le I_j \le I_j^{max}, \quad \forall j. \\ & \bar{\bar{A}}^{n|n-1} = (\bar{K}^n)^T\bar{K}^n\bar{\bar{G}}^T\bar{\bar{P}}^{n|n-1}\bar{\bar{G}}, \\ & \bar{b} = -\bar{\bar{G}}^T\bar{\bar{P}}^{n|n-1}\bar{K}^n. \end{aligned} \quad (7)$$

Here, $\bar{\bar{G}}$ is a $V \times S$ matrix that transforms the source illumination vector $\bar{I}^n$ to the weight matrix $\bar{W}^n = (\bar{\bar{G}}^n.\bar{I}^n)^T$. The optimization problem in (7) is a convex Quadratic Programming (QP) problem with inequality constraints and has a single (global) minimum. Thus, to find the minimum of (5), we repeat the computation of $\bar{K}^{*n}$ and $\bar{I}^{*n}$ in an iterative loop until convergence is achieved. We then illuminate the sample with the calculated optimal illumination pattern to make the measurement. The result of the measurement is then used to update our estimation of $\bar{\eta}$ and its covariance matrix.

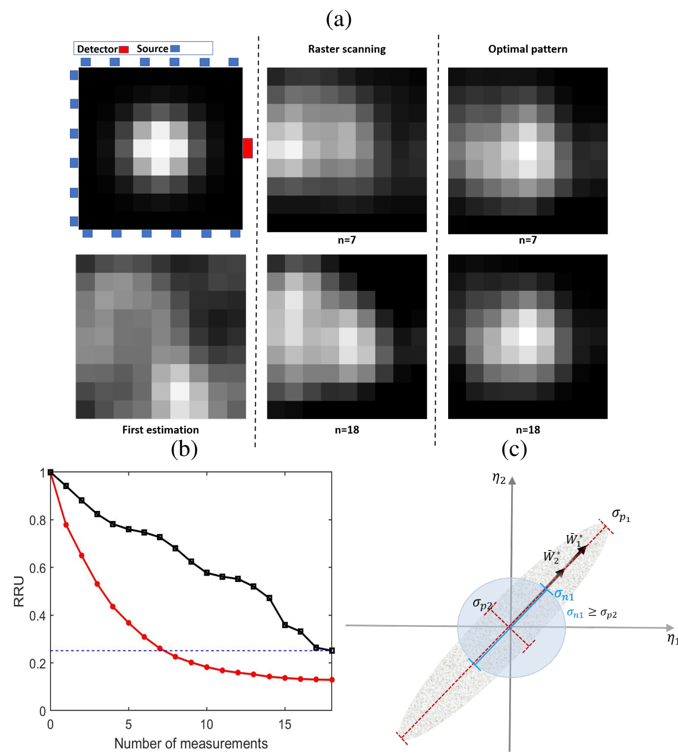

Fig. 1: (a) Structure of a 2D square single-detector scanner and the comparison between optimal illumination and raster scanning. Following optimal illumination algorithm, one can reconstruct images with acceptable distortion while taking a smaller number of measurements, (b) RRU values plotted as a function of measurement number for two different scanning protocols, (c) distribution of noise and the scattering parameter along with the principal components of the data.

To better study the performance of our algorithm, a simple simulation experiment was carried out by modeling a 2D single-detector scanner shown in Figure 1a. The scanner is square in shape, with 18 sources evenly spaced on three sides. 81 pixels were used to discretize the region within the scanner. Scattering potentials of these pixels are unknown variables for which we have prior estimation of their mean values and the corresponding covariance matrix. This limited number of pixels was only used to illustrate the concept. We simulated 18 rounds of optimal measurements/updates. For comparison, reconstructed images generated by the raster scanning and optimal pattern protocols are shown at two different stages in Figure 1a. The actual distribution and our first estimation are also shown in the first column. Even though the first estimation is significantly different from the actual distribution, our proposed algorithm was able to predict the distribution with a reasonable level of accuracy after 18 measurements. Also, it can be seen that our optimal illumination method outperforms raster scanning even after making seven measurements. The trace of the covariance matrix was computed after each update and normalized by the initial trace value to calculate the Relative Residual Uncertainty (RRU). Curves in Figure 1b show the evolution of RRU values when the scanner was following the optimal illumination pattern algorithm as well as the conventional raster scanning. Notice that raster scanning reaches a certain level of certainty after 18 measurements, while optimal illumination method reaches the same level of certainty after making only 7 measurements. This data proves that the optimal illumination approach, compared to raster scanning, expedites the scanning process and generates more accurate images while taking a smaller number of measurements.

Since every feasible illumination pattern is a linear combination of patterns produced in raster scanning, we expect the final value of RRU obtained by raster scanning to be equal to or better than what is eventually obtained by optimal patterns. However, as it is shown in Figure 1b, in the presence of measurement noise, optimal illumination algorithm leads to a smaller final RRU value. To better understand this effect, consider a simple 2D single-detector scanner with only two sources and two pixels. Based on the theory of principle components, maximum uncertainty occurs along eigenvectors of the covariance matrix. In this figure, the estimation vector $\bar{\eta}$ is a 2D vector and therefore we have only two principal components. Our proposed method suggests that each measurement should minimize the uncertainty left in the estimation as much as possible. In other words, when there is no constraint on source intensities, the algorithm designs a measurement vector that is along with the largest principal component of the estimation vector. After we make the measurement, we update the covariance matrix using (4) to search for the next illumination pattern. Note that the covariance matrix update equation (4) consists of two parts. The first part is the amount of uncertainty left in other principal components and the second part, the term $K^n\bar{\bar{R}}(K^n)^T$, is the noise uncertainty projected on the direction of current measurement(largest principle component). If this noise component is larger than the next largest principal component of the data, the same measurement is repeated to reduce the effect of noise. This denoising effect, as seen in Figure 1b, enhances the optimal illumination algorithm's performance in terms of scanning speed and ultimate RRU value. This is a key fact that other scanning techniques, such as raster scanning, overlook.

B. Multi-Detectors Scanner

In this section, the problem is seen from a more general point of view which is multi-detectors scanner. Note that, in this case, $\bar{\bar{W}}$ is a $D \times V$ matrix and noise uncertainty is represented by noise covariance matrix $\bar{\bar{R}}$. Once again, we start the optimization by assuming that the weight matrix, $\bar{\bar{W}}^n$, is constant and we minimize $Tr(\bar{\bar{P}}^{n|n})$ with respect to Kalman gain.

$$\bar{\bar{K}}^{*n} = \bar{\bar{P}}^{n|n-1}(\bar{\bar{W}}^n)^T \left[\bar{\bar{W}}^n \bar{\bar{P}}^{n|n-1}(\bar{\bar{W}}^n)^T + \bar{\bar{R}}\right]^{-1}. \tag{8}$$

In a multi-detectors scanner, determining the best illumination pattern results in the following form:

$$\begin{aligned} \bar{I}^{*n} = \arg\min_{\bar{I}^n} \; & -2Tr\left[\bar{\bar{K}}^n \bar{\bar{W}}^{(n)} \bar{\bar{P}}^{n|n-1}\right] \\ & +Tr\left[(\bar{\bar{K}}^n \bar{\bar{W}}^n)\bar{\bar{P}}^{n|n-1}(\bar{\bar{K}}^n \bar{\bar{W}}^n)^T\right], \\ s.t. \qquad & I_j^{min} \le I_j \le I_j^{max}, \quad \forall j. \end{aligned} \tag{9}$$

where:
$$\bar{\bar{W}}^n = \bar{\bar{G}}_2 \cdot diag(\bar{\bar{G}}_1 \cdot \bar{I}^n). \tag{10}$$

The objective to be minimized in equation (9) is not convex and therefore, finding the global optimum is not an easy task. However, since the objective function is differentiable with respect to $\bar{I}^{(n)}$, we propose an algorithm to guide us to an approximately optimal solution. We use a version of the gradient descent method that can accommodate the inequality constraint as well. The Projected Gradient Descent (PGD) method is well-suited to this constrained optimization problem, see figure 2.

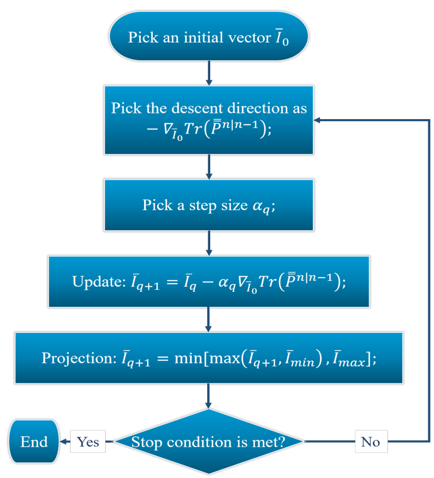

Fig. 2: Optimal pattern algorithm in multi-detectors scanner.

Once the solution to problem (9) is found, we use equation (10) to calculate the optimal weight matrix, $\bar{\bar{W}}^{*n}$. Then, we assume $\bar{\bar{W}}^n$ is constant to compute the optimal Kalman gain using equation (8). We continue till convergence.

To evaluate the performance of our method on a multi-detectors scanner, we conducted simulations using an $8cm^3$ cubic phantom with two cylinders inside, located with an edge-to-edge distance of $0.6cm$ and filled with fluorescent agent. The cube and cylinders had the same height of $2cm$ and the diameter of cylinder was set to $0.4cm$. 8000 voxels

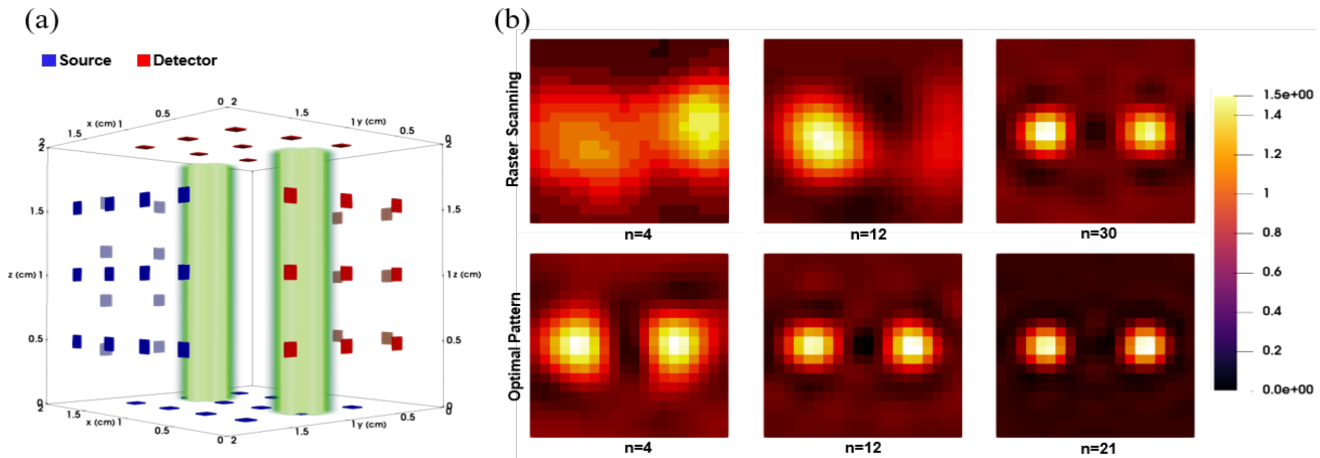

Fig. 3: (a) Cubic phantom simulation setup, (b) cross-sections of fluorescence targets reconstructed via raster scanning (top row) and optimal pattern method (bottom row) shown for different number of measurements.

were used to evenly mesh the phantom. We performed the simulation with 36 sources and 27 detectors mounted on opposing walls of the cubic scanner as shown in Figure 3a. Optimal illumination pattern was obtained for each measurement and the estimation was updated after each observation. Raster scanning was also performed for comparison. Figure 3b shows the evolution of reconstructed cross-sections at the depth of $z = 14mm$ for both optimal illumination pattern and raster scanning for different number of measurements. Results show clearly that the optimal illumination algorithm outperforms raster scanning. The two targets could be identified after making 12 measurements using optimal patterns, while raster scanning required 30 measurements to reconstruct images of fluorescence objects with acceptable quality.

III. Discussion and Conclusions

Data acquisition time is a determining factor in some medical applications of tomography. Additionally, In certain types of tomography such as X-ray imaging or where radioactive materials are used, the sample is exposed to several doses of radiation while scanning, which can be harmful. Therefore, in this study, we developed a systematic approach for optimizing the scanning process by reducing the number of measurements required to achieve satisfactory image quality. In the past, Fisher information has been used for design of experiment. However, it only provides partial optimization. In the case that there is no prior information and detectors' noises are independent, the Kalman method proposed in this study converges to Fisher information approach. In the proposed framework, a definition for the achievable resolution of the scanner can be provided, in which effective parameters in improving image quality can be identified. Then, we can show how our method incorporates these parameters, e.g., scanner geometry and prior information, as resolution enhancing factors. The proposed method also opens up the possibility of optimizing the geometry of the scanner or the location of sources/detectors to achieve superior efficiency. This could be done by imposing additional constraints to the optimization problem so that during each measurement optimal location of sources/detectors are selected among all possible pre-defined locations. In addition, the probabilistic approach allows for the definition and incorporation of various sources of uncertainty (e.g., state-dependent noise) that cannot be separately identified in other deterministic approaches. The efficiency of current work is even more significant in the scanning of dynamically evolving objects which will be the focus of future studies.

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