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Image Super-Resolution Through Compressive Sensing-based Recovery

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***Abstract*— The primary aim of image super-resolution techniques is to produce a high resolution (HR) image from a low resolution (LR) image efficiently. Deep learning algorithms are being extensively used to address the ill-posed problem of single image super-resolution which requires extremely large data-sets and high processing power. When one does not have access to large data-sets or have limited processing power, an alternative technique may be in order. In this study, we have developed a novel positive scale image resizing method inspired by compressive sensing (CS). We have considered the image super-resolution as a CS recovery problem in which a low resolution image is assumed as a compressed measurement and the required interpolated image is treated as output of the CS-based recovery. In the proposed HR recovery method, a deterministic binary block diagonal measurement matrix, (DBBD), is used as measurement matrix since it maintains the visual similarity between the low and high resolution images. Then along with a sparsification matrix, the sparse representation of HR image is first recovered and subsequently the dense HR image is obtained. The proposed method is applied to medical and non-medical images. The HR images obtained using the traditional proximal, bilinear and bi-cubic interpolation techniques are compared with those obtained using the proposed method. The proposed CS inspired method delivers superior HR images than the traditional techniques. The superiority of the proposed method is attributed to the unique usage of the DBBD matrix and the CS recovery algorithm to obtain a high resolution image without any prior training data-set.**

***Index Terms*–Compressive Sensing, Image interpolation, Image Super-Resolution, Deterministic sensing matrix, Recovery techniques**

I. Introduction

Images with high resolution help in making better diagnostic decisions from medical images. Often an image with higher resolution (HR) will enable better detection of anomalies such as tumors and cancerous cells than a low resolution (LR) image. The quality and the resolution of an image obtained through various medical imaging systems such as X-rays, magnetic resonance imaging or computer tomography play a crucial role in the diagnosis of a disease. With high resolution images, it is possible to design automatic diagnostic tools that could aid medical professionals to make accurate decisions. It also enables doing object detection and image segmentation with higher accuracy [3]. Realizing this, many diagnostic tools based on deep learning models [1], [2] have been proposed lately. All these models, however, require lot of images for training purposes. When only one low resolution image is available, no learning model can be developed. This paper aims to perform super-resolution (SR) from just a single low resolution (LR) image under the assumption that there is no access to any huge dataset.

The term SR can be defined as a technique to enhance or increase the resolution of an image. Single image super-resolution (SISR) aims to generate a HR image from a LR one. Traditionally, super-resolution is attempted either using multiple images and solving for a set of linear constraints or by learning relationship between LR and HR image patches from a database (called example-based approach) [4]. Since the mapping between LR image and HR images are not unique, SISR is an ill-posed problem for image recovery [5]. Although multiple HR images were used in [6] to reconstruct the HR image, the reconstructed image was not guaranteed to contain true HR details. Since recovery of HR was based on multiple examples, new learning algorithms such Bayesian approach [7], neighbour embedding method [8], recovery using sparse patches from LR images [9] were introduced. Over the years, interestingly even for SISR problem, deep learning (DL)-based approaches have become the most sought method.

Amongst the DL-based approaches, the SR convolutional neural network (CNN) has become the benchmark architecture for DL-based SR algorithm [10]. Deep neural network based unsupervised algorithms such as the deep Boltzmann machine [11], variational autoencoder (VAE) [12] and generative adversarial nets (GAN) [13] have also been implemented to handle unlabeled data situations. All these methods claim their superiority in terms of accuracy but do not highlight the shortcomings associated with them. Firstly, DL algorithms need large pre-trained data-sets for efficient mapping (or learning) which may not be available in many problems. Secondly, DL techniques are image specific. Thirdly, the DL networks may have the problem of over-fitting. Particularly for medical images, such erroneous results may lead to wrong diagnosis. Lastly, implementation of DL algorithms are computationally intensive and therefore require computers with huge processing capability. Unfortunately, DL methods are of no avail when only one LR image is available and needs to be converted into a SR image.

To overcome the aforementioned shortcomings, non-learning based algorithms may be used. Traditional prediction models or interpolation techniques such as proximal, bilinear or bicubic interpolation use weighted average neighbouring LR pixel intensities to generate a HR image. The major drawback of these techniques is that they generate a smoother version of the HR image losing large gradients along the edges and at high frequency regions.

Learning-based approaches were used in CS for SR applications in [14], [15]. A learning-based CS recovery method utilizing sparsity of HR image in wavelet domain and its recovery using greedy algorithm was introduced in [16]. Later, an analogy between CS and SR was drawn to demonstrate a better understanding of the role of sparsity priors and the properties of the projection operators and dictionaries [17], [18]. In this paper, we address the problem of SISR and present a unique non-learning based approach to obtain the HR image through CS-based recovery with just a single image.

A deterministic binary block diagonal (DBBD) matrix that preserves the structural similarity between the LR and HR images in the recovery process is used in this work. Instead of recovering the HR image directly, first a sparse representation of the HR image is recovered. Discrete cosine transform (DCT)-domain is assumed to sparsify the considered images. A fast CS recovery algorithms, smoothed ℓ_0 (SL0) [19], in order to reconstruct a HR image is implemented. A comparative study between the various traditional interpolation techniques and the proposed technique using objective quantitative measures like peak signal-to-noise ratio (PSNR), mean square error (MSE) and structural similarity index (SSIM) is presented. Since there is no pre-trained data-set or learning involved, the proposed SR approach is computationally inexpensive. Moreover, the use of CS-based recovery helps in preserving the gradients at high frequencies. Thus, this proposed method successfully overcomes the drawbacks of the DL-based and the interpolation-based approaches. The idea of this paper is to recover a HR image from a single LR image without any external information.

The paper is organized in the following manner. Section II gives a brief overview of compressed sensing and its recovery, measurement and sparsification matrices.Section III contains the proposed method, Section IV provides the results, Section V presents the discussion while Section VI concludes the paper.

II. COMPRESSIVE SENSING

CS is a sampling technique that enables reconstruction of sparse signals which are sampled well below Nyquist rate. In this section, both 1-D CS and 2-D CS principles are discussed briefly.

A. 1-D compressive sensing

CS involves compression and recovery of sparse signals in some known basis. A sensing or measurement matrix is the matrix that transforms a sparse vector from a higher dimension to a lower dimension. The sensing matrix is generally a random matrix that results in a compressed vector that does not preserve any similarity between the input and the compressed samples. Thus to overcome this, deterministic matrices can be used as sensing matrix. Deterministic matrices preserve the structure of the input vector.

Considering a K-sparse signal, $\mathbf{x} \in \mathbb{R}^n$ that needs to be compressed. Let $\mathbf{\Phi}$ be a $m \times n$ sensing matrix, where $m << n$. [20], [21]. Let $\mathbf{y} \in \mathbb{R}^m$ be the measurement vector given by:

$$\mathbf{y} = \mathbf{\Phi\Psi s} \tag{1}$$

where the sparse vector $\mathbf{s}$ is given by $\mathbf{s} = \mathbf{\Psi}^T\mathbf{x}$ and $\mathbf{\Psi}^T$ is an orthonormal matrix which sparsifies the signal $\mathbf{x}$ and T is the transposition operator.

Since the above system is under determined, the problem of recovery is ill-posed and has infinite solutions but due to the sparse representation of $\mathbf{x}$ with respect to $\mathbf{\Psi}$, the recovery can be done by finding the sparse vector $\hat{\mathrm{s}}$ through l_0-minimization:

$$\begin{aligned} \hat{s} &= \min_{\mathbf{s}} ||\mathbf{s}||_0 \\ & st. \quad \mathbf{y} = \mathbf{\Phi\Psi s} \end{aligned} \tag{2}$$

Now the signal $\hat{\mathbf{x}}$ can be reconstructed from the estimated $\hat{\mathrm{s}}$ using the following equation:

$$\hat{\mathbf{x}} = \mathbf{\Psi}\hat{\mathbf{s}} \tag{3}$$

In order to obtain the sparse solution, the sensing matrix, $\mathbf{\Phi}$, should have Restricted Isometric Property (RIP) [22], [23], [24]. Since RIP is difficult to confirm, mutual coherence between the sensing matrix, $\mathbf{\Phi}$, and the sparsification matrix, $\mathbf{\Psi}$, may be used.The mutual coherence between these matrices should be very low for a good recovery. Random matrices satisfy both these properties but the compressed measurement and and the input signal are visually different. Recently, in [25], a deterministic matrix called Deterministic Binary Block Diagonal (DBBD) matrix was proposed. This matrix ensured that the compressed measurement was visually similar to the input signal.

Deterministic Binary Block Diagonal matrix (DBBD) is a unique binary matrix with values of 1 only along the diagonal.Following is an example of Deterministic Binary Block Diagonal (DBBD) sensing matrix ($\Phi_{m\times n}$) in small size ($m = 4$ rows and $n = 8$ columns)

$$\mathbf{\Phi}_{m\times n} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{4}$$

Diagonal 'blocks' of this matrix containing $\boldsymbol{r = \frac{n}{m} = \frac{8}{4} = 2}$, 1s and the rest are zero. The mutual coherence of this matrix with the DCT dictionary is lower than other sensing matrices.

B. 2-D Compressive Sensing

Consider a 2-D signal, $\mathbf{X} \in \mathbb{R}^{\boldsymbol{n}\times\boldsymbol{n}}$, where $\boldsymbol{n}$ is a large number. CS for this image can either be block-based [26] or column-based following the 1-D CS paradigm. Block-based sensing leads to a computationally intensive recovery process and also requires the identification of sparse blocks during the sensing process. Unfortunately, often sparse blocks are not known a-priori and thus even the sensing process becomes computationally challenging. In [27], it was shown that each column of the image could be compressed and recovered individually following 1-D CS. The work in [28] extended the column CS idea to the rows of the image and applied CS on both, columns and rows, to obtain a compressed image following the work in [29]. Without loss of generality, if we suppose the same measurement matrix is applied to the both rows and columns of an image, the row-column wise sensing of an image can be obtained as follows:

$$\mathbf{Y}_{m\times m} = \mathbf{\Phi}_{m\times n} \times [(\mathbf{\Phi}_{m\times n} \times \mathbf{X}_{n\times n})]^T \quad (5)$$

where superscript T stands for matrix transpose. Equation 5 shows that this model equals to applying CS two times as follows:

$$Z_{m\times n} = \mathbf{\Phi}_{m\times n} \times X_{n\times n} \quad (6)$$

$$Y_{m\times m} = \mathbf{\Phi}_{m\times n} \times (Z_{m\times n})^T \quad (7)$$

In the above equation, the column compressed output of the original image, $Z_{m\times n}$, is transposed to compress the second time CS. This process is shown in Fig. 1.

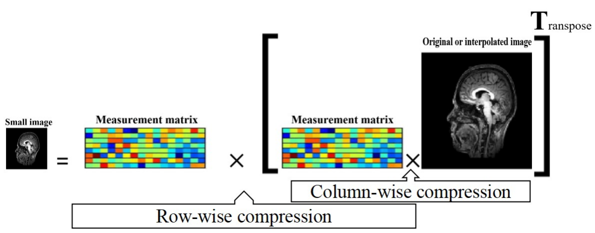

Fig. 1. Compression Stage

III. PROPOSED SUPER-RESOLUTION METHOD

In image super-resolution, there is a high visual similarity between the LR and the corresponding HR images. From Section II, $\mathbf{X}$ is the original HR image while $\mathbf{Y}$ is the LR image. If the visual similarity between HR and LR images needs to be maintained, then the measurement matrix cannot be a random matrix and should be a deterministic matrix. It was reported in [26], [27] that DBBD measurement matrix maintains the visual structure between $\mathbf{Y}$ and $\mathbf{X}$. In this work, we suppose the given LR image was obtained by 2D CS compression where the HR image was sensed using DBBD matrix. Therefore, the SR problem is viewed as a 2-D CS recovery (inverse problem), i.e. obtaining X given Y and $\mathbf{\Phi}_{DBBD}$.

A. 2-D CS Recovery

Since we suppose the X (HR image) has been compressed column-wise and row-wise, we apply recovery in two steps to obtain the HR image. For simplicity, let us suppose $A_{m\times n} = \mathbf{\Phi}_{m\times n} \times \psi_{n\times n}$. In the first recovery stage, we reconstruct the approximately sparse matrix $\hat{S}_{n\times m}$ (sparse with respect to the columns, i.e. each column $\hat{s}_i$ is approximately sparse) with the help of the following l_1-minimization problem:

$$\min_{\hat{s}_i \in R^{n\times 1}} \|\hat{s}_i\|_1 \quad s.t \quad \|y_i - A\hat{s}_i\|_2 < \epsilon \;\; i = 1, 2, ..., m \quad (8)$$

$\hat{S}$ contains m approximately sparse vectors, and $Z_{n\times m} \approx \hat{Z}_{n\times m} = \psi_{n\times n} \times \hat{S}_{n\times m}$. $\hat{Z}$ can be assumed as interpolated image along its columns. We apply the recovery second time to reconstruct the interpolated image along its rows as well. In this case $\hat{Z}$ transposed is used as measurement for the second recovery as follow:

$$\min_{s_i \in R^{n\times 1}} \|s_i\|_1 \quad s.t \quad \|\hat{z}_i - As_i\|_2 < \epsilon \;\; i = 1, 2, ..., n \quad (9)$$

where $\hat{z}_i$ is the ith row of the $\hat{Z}$ and ϵ is a user defined threshold. After the recovery, S contains n approximately sparse vectors, and $X_{n\times n} \approx \hat{X}_{n\times n} = \psi_{n\times n} \times S_{n\times n}$. $\hat{X}$ is the final HR image which has been interpolated along its columns and rows.

IV. SIMULATION RESULTS

In this work, we performed our simulations with standard test images available in MATLAB as well as with Magnetic Resonance Imaging (MRI) brain images mentioned in [27]. The procedure adopted there in to obtain the 2-D images was followed. As mentioned in [27], each case contained 90 3-D slices. Each 3-D data sequence of 100ms interval was further sliced to 2-D images of size 256×256 using the slicer program available along with database. We chose randomly 3 slices, from three randomly chosen cases of 2-D images, to conduct our evaluation. The code that we developed is available for download from [30]. The LR images for this study were obtained using "resize" command of MATLAB on the original HR images available in MATLAB and the database. Then we applied our proposed SR method, and also the standard SR methods such bicubic, bilinear, and nearest neighbour to compare the performance of our proposed method. For the proposed method, we used DBBD matrix as measurement matrix and DCT dictionary as sparsifying basis. SL0 method was used to obtain the sparse matrix and with the DCT dictionary, the original HR image was obtained.

To evaluate the quality of obtained HR images, their corresponding ground truth images were used and metrics such as the peak signal-to-noise ratio (PSNR) and mean squared error(MSE) were calculated. The following equations for PSNR and MSE were used:

$$MSE = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}(X_{ij} - \hat{X}_{ij})^2$$
$$PSNR = 10\log_{10}\frac{R^2}{MSE} \quad (10)$$

where X_{ij} is the pixel of ith row and jth column of original HR image (ground truth) and $\hat{X}_{ij}$ is the corresponding pixel of the HR image obtained using our proposed and standard SR methods. R is the maximum possible pixel value of the obtained HR image.

We also measured the structural similarity using structural similarity index between the original and output image to further affirm the similarity [31]. SSIM is a multiplicative combination of luminance, contrast and structure. Mathematically, it can be written as:

$$SSIM = \frac{(2\mu_x\mu_y + c_1)(2\sigma_x y + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \qquad (11)$$

where μ_x and μ_y are the means of x and y respectively, σ_x^2 and σ_y^2 are the variances of x and y respectively and c_1 and c_2 are two variables to stabilize the denominator. Our simulation results show that proposed model outperforms standard methods of image interpolation in terms of output PSNR, MSE, and SSIM indices.

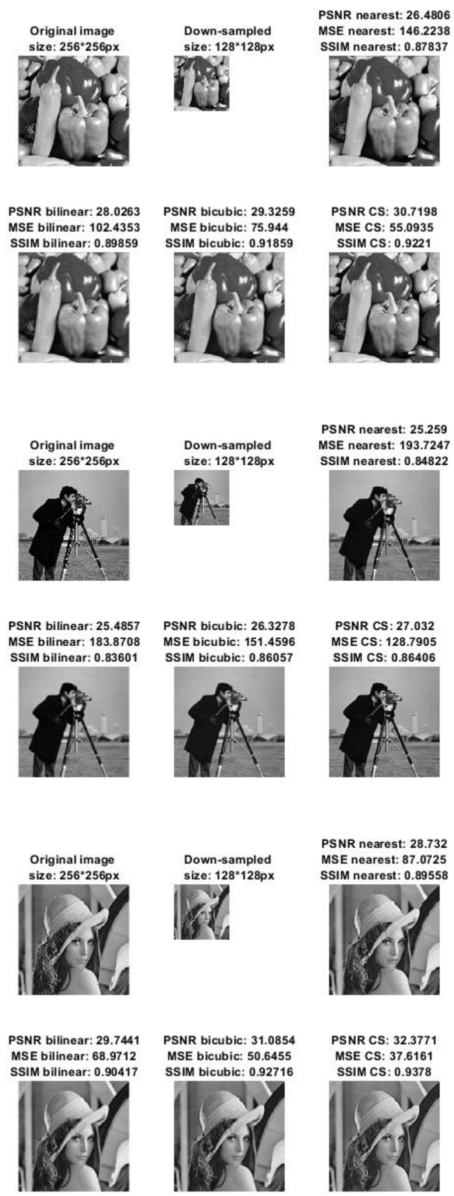

Fig. 2. Simulation results of standard test images

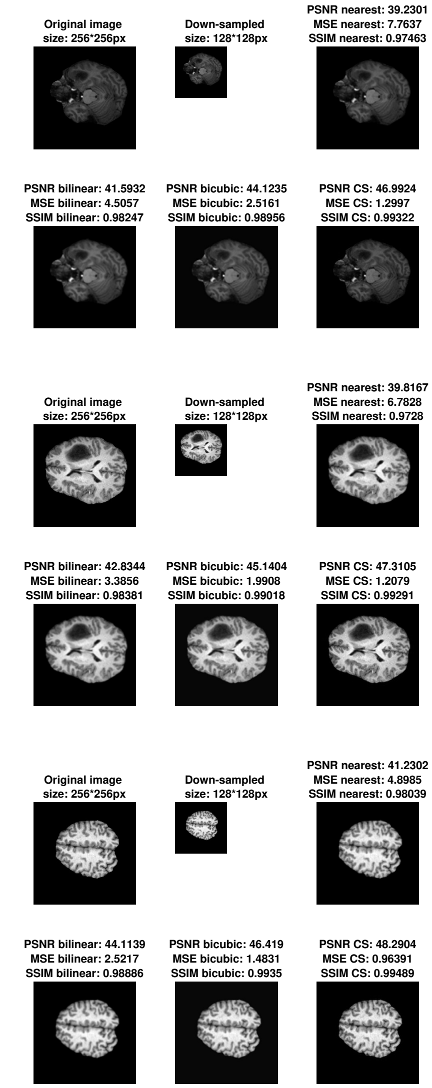

Fig. 3. Simulation results of MRI images

V. DISCUSSION

In Fig. 2 and Fig. 3, it is observed that our method results in high PSNR value, low MSE value and SSIM value closer to 1 when compared to other standard methods. High PSNR value indicates that the image recovered has less noise whereas low MSE value shows greater similarity between the original image and the reconstructed image. SSIM close to 1 indicates high structural similarity of the interpolated image with the original image. Our method successfully recovered the image even though there was a high loss of information due low-pass filter attenuating some of the high frequency components. However, an image with a large number of high frequency components or extremely low resolution (below 128×128) may lose some amount of information while sensing.

VI. CONCLUSION

In this paper, we have proposed a novel method to resize input images through compressed sensing recovery. We have used SR in framework of compressed sensing, that is, conversion of a low resolution image to a high resolution image using compressed recovery technique. Our model uses deterministic matrices as sensing matrices which makes recovery efficient and accurate. It is confirmed from the simulation results that this method outperforms the standard image resizing techniques like the nearest neighbour, bilinear and bi-cubic in terms of objective measurement and visual quality. The high value of PSNR, the low MSE value and SSIM close to 1 indicates a good recovered high resolution image with edges well-preserved. We have demonstrated that the proposed SR technique is effective for both regular and medical images.

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