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On the Information Theory for Magnetic Resonance Imaging

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Abstract— **This work presents the mathematical formulation of the Magnetic Resonance Imaging (MRI) system modeled as a wireless communication system to establish its information theory foundations. The MRI system conceived as a source-sink communication system has channel impairments that affect the transmitted data. The information source is a stochastic process that produces a sequence of information symbols governed by a set of probabilities. The adverse effects on the transmitted MRI signal shall limit the amount of information capable of being received at the sink. Therefore, reliable detection at the receiver shall be accomplished by estimating the channel capacity and an approximation of the source entropy. Modeling the MRI system using a wireless model shall simplify the receiver architecture, yielding new methods to improve MRI signal acquisition, i.e., different values of bandwidth and signal strength yield the same channel capacity. Achieving capacity bridges information and computation efficiency.**

Clinical relevance **This work establishes the basics to reduce MRI scan times by acquiring sufficient information with less redundant information.**

I. INTRODUCTION

Multiple studies have been developed to improve MRI acquisition methods, including Nyquist sampling rate estimate and ever complex image reconstruction techniques, without considering the source of information as a stochastic process. These efforts have the ultimate goal to improve the image pixels, the latter being the central figure of merit to assess success. This figure of merit is ambiguous since it varies from person to person, and its top performance value reaches the limit of the human vision constraint. Thus, the mission to measure how much information can be extracted from the signal source has been unnoticed. This work first asks: what is the channel information limit? Meaning, how much data the system requires to reconstruct the information uniquely from the body to the coil receiver with high probability. For instance, under a specific noisy channel, only a limited amount of information can be extracted, independent of the methods used to acquire the data at the receiver.

From information theory [2], [3] it is stated that a single number, its *capacity* characterize the channel. If the information rate of a source model is less than the channel capacity, then it can be transmitted virtually error-free over the channel by doing appropriate processing. Therefore, there is a reasonable assumption that all information encapsulated in the MRI waveform can be fully recovered since the channel is the free space without spectrum resource constraints.

Shannon's statement in [2], [3] reveals a unifying theory with profound intersections with Probability, Statistics, among other fields, to set the stage for the development of data storage and processing (e.g., data compression techniques), communications among other technologies. Shannon's work states the communication theory encompassing the fundamental tradeoffs of transmission rate, bandwidth, reliability, and signal-to-noise ratio. The channel bandwidth and signal power strength are the primary communication resources.

A. Information Theory

A general communication system has three main components, the source of information, the channel where the information travels through, and the sink where the information is received. The main idea is to reproduce precisely the information from source to destination. "The actual received message is one selected from a set of possible messages" [2], [3], and the source of information resembles a random process.

1) Definitions: Some definitions are needed to describe the communication system's functionality.

Message: Amount of transmitted data or symbols from the source.

Information: Part of the message which is new to the sink.

Redundancy: Difference of message and information, which is unknown to the sink.

Message = Information + Redundancy

Irrelevance: Information that is not essential to the sink (not originated from the source).

Equivocation: Information not steaming from the sink of interest.

II. INFORMATION THEORY FOR MRI SIGNAL SOURCES

Fundamentally, an MRI system is a wireless communication system. The wireless MRI communication system includes the information source, a channel model, and a sink component as the information destination. Fig. 1 shows the wireless MRI communications system model.

The information source produces a sequence of symbols weighted with individual probabilities yielding a stochastic process model. Without losing generality, the information source includes radio frequency (RF) pulses and gradients

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$G(r,t)$ that affect the state of the human body under the stress of a strong static magnetic field[1] $\mathbf{B}_0(F)$ and $\mathbf{B}_1(F)$.

The channel adds Additive white Gaussian noise (AWGN) to the signal as it passes through it. The channel model represents the space the signal x travels from the human body to the MRI coil. The channel is modeled as a filter with an impulse response $\mathbf{h} = [h_0.h_1, \dots, h_{l-1}]$ that affects the signal composition. This channel is noisy because the inputs do not determine their outputs but instead have only a stochastic dependence on the input. Thus, given any input sequence, the output sequence is a stochastic process with a known distribution conditional to the input. Furthermore, the channel input sequence is arbitrary. Choosing the encoding relationship between the source output and the channel input is the most critical degree of freedom in designing a system for reliable communication. The channel input and output may be described as a joint stochastic process once one knows how this source/channel input processing behaves. Finally, the information arrives at the information sink, where the signal is post-processed.

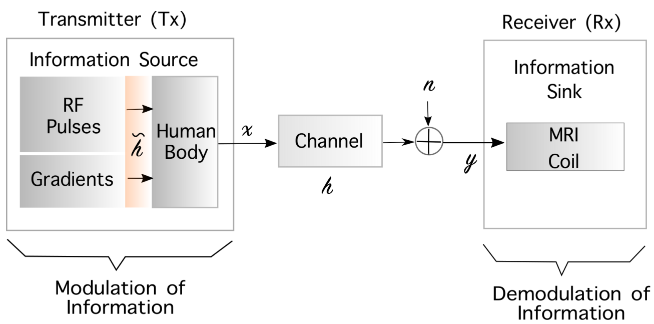

Fig. 1. MRI Communications System Model: Source-Sink

To formulate the requirements at which the signal needs to be fully reproduced at the sink, one needs to model the information source rate at which the data can be transported from source to destination without errors, in other words, its channel capacity.

A. MRI Information Source - Transmitter (Tx)

The MRI information source nature resembles a continuous waveform. However, it can be considered as the construction of digital messages, each composed out of M different symbols, such that it is a digital source of messages limited to a set of possible outputs, an alphabet $\mathscr{A}$. Thus, the alphabet to adequately represent the signal tends to infinity $\mathscr{A} \to \infty$, but finite. In practice, the alphabet definition is defined as $\mathscr{A} = \{s_1, s_2, \dots, s_{l-1}, s_l\}$, where l it is an integer representing the number of elements in the set $\mathscr{A}$. The human body represents a system with an output impulse response. The system is a multiple-input-multiple-output (MIMO) system where the inputs are the RF pulses, the gradients, and the strong magnetic field B_0. The inputs to the human body system are affected by the free-space channel $\tilde{h}$ such that before the inputs arrive at the human body, they are modified, i.e., $(RFPulses + Gradients) * \tilde{h}$. The above process is referred to as "*Pulse Sequence Design*," not treated in this text. Instead, the stochastic nature of the signal source, x, is accounted. Any stochastic process that generates a discrete sequence of symbols selected from a finite set shall be considered a discrete source [4]. The complex signal output x represents the information from the impulse response of the human body to the multiple inputs such that $x \in \mathscr{A}$. Note that x is a limited bandwidth signal.

Modeling the impulse response output of the human body might be a complicated task; fortunately, due to the nature of the human body, its impulse response is limited in frequency and bandwidth. For the MRI general studies, the frequencies of interest relate to the water and fat molecules. Also, there are the so-called *"chemical shifts"* responses that sit on top of the water and fat spectrums. As such, a complex signal x defines the information source as:

$$\begin{aligned} x &= (RF_p + G(r,t)) * \tilde{h} + n \\ &= \tilde{x} * \tilde{h} + n \end{aligned} \tag{1}$$

where $\tilde{x}$ represents the sum of all input signals (RF pulses plus gradients), $\tilde{h}$ being the free-space channel impulse response inside the MRI room, and n the additive white Gaussian noise. The $*$ sign stands for the convolution operation.

Furthermore, modeling x as a multi-carrier signal, where the information is not present at a single frequency, but in a range of interest, the bandwidth $\mathscr{B}$. Interestingly, for every frequency range there might exist an impulse shaping $g_{T_x}(t)$ in $\tilde{h}$ that affects a portion of the signal x. For completeness, there might be a shaping filter at the receiver (sink) $g_{R_x}(t)$, such that the signal can be detected, acting as a matching filter, with the upcoming symbols y. Thus, the presence of an equalizer at the receiver can compensate for the channel SNR losses due to fading across the bandwidth, see [12] for reference.

Fundamental physical quantities influence the information source in the MRI environment are: Larmor Frequency (1.5T) -¿ 63.85MHz, $\delta f =$ Frequency offset for slice location, $T =$ Pulse width (seconds), $\Delta f =$ RF Bandwidth (Hertz), $T\Delta f =$ Dimensionless *time-bandwidth product*, measure of the selectivity of the pulse and determined by the pulse shape [8], $\phi = \gamma B_1 T = \textit{flip angle}$ (degrees), describes the nutation angle produced by the pulse, $SAR \approx B_0^2, \phi^2 \Delta f =$ Specific Absorption Rate (SAR - Watt per Kg.) describes the unwanted heating in patient's tissue, $\gamma =$ Gyromagnetic ratio (radians/s/tesla), $\phi(t) = \gamma \int_{t'=0}^{T} B_1(t')dt' = \textit{flip angle}$ produced on-resonance.

a) SINC Pulse Bandwidth: The Fig. 1 shows the RF pulses used as input to the human body. Different pulses can be used to complete an MRI scan. For example, if a SINC pulse is used the approximate bandwidth equals to:

$$\Delta f \approx \frac{1}{t_0}, \quad \text{where} \quad \Delta f \in \mathscr{B} \tag{2}$$

being $\mathscr{B}$ the total bandwidth at the receiver.

[1]Normally $\mathbf{B}_0$ field strength depends on the installed system, e.g., 1.5T and 3.0T

b) Pulses' Bandwidth: Variable-rate (VR) pulses are also known as variable-rate gradient (VRG) pulses or variable-rate selective excitation (VERSE) pulses. One of the primary applications of the VR pulses is to reduce the RF power deposition to the patient; the reduction is accomplished by decreasing the RF amplitude in the vicinity of the peak of the pulse [8]. Another use of the VR pulses is to direct the gradient ramps to allow techniques for slice selection (thinner slices) to improve the slice profile.

In general, the VR pulses generate a variation on the actual bandwidth used proportional to the combination of pulses' shapes. Thus, this fact reinforces the approach to treat x (see Fig. 1) as a multi-carrier signal that works on a confined space in the frequency domain (bandwidth length), with the characteristic that not all bands are active all the time as shown in Fig. 2. Furthermore, the bandwidth portion equals to: $f_i - f_{i-1} = \Delta f$, as described in equation (2).

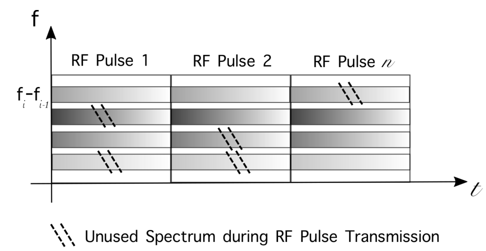

Fig. 2. Generalized information source as a multi-carrier system in the MRI communication system model. The information spans a range of frequencies, e.g., for the 1.5T MRI system, the *Larmor* frequency is different from the 3.0T system, and for spectroscopy applications, several frequencies are of interest.

Composite Pulses are meant for frequency selectivity, targeting a dedicated chemical component, i.e., water and not fat, see Fig. 2. To describe the nature of those pulses it follows:

$$S_j(f) = \cos^{\rho}\ (\pi f \tau) \tag{3}$$

where f describes the frequency of interest, τ stands for time intervals between centers of two adjacent pulses, and ρ it is an integer pointing to a specific pulse component. Furthermore, the frequency envelope is described by the Fourier transform of $S_\rho(f)$ and directly proportional to the binomial coefficient the pulse sequences:

$$\mathscr{F}\left[S_\rho(f)\right] \ = \ \sum_{k=0}^{\rho} q_{\rho,k}\ \delta\left(t - \frac{\rho\tau}{2} + k\tau\right) \tag{4}$$

where the binomial coefficients $q_{\rho,k}$ are:

$$q_{\rho,m} \ = \ \binom{\rho}{m} \ = \ \frac{\rho!}{(\rho-m)!\ m!}. \tag{5}$$

Having the off-resonance frequency $f = \frac{1}{2\tau}$, if $\tau \gg T_1\ or\ T_2$, relaxation effects are negligible. If τ is chosen such that $f = 0$ and f targets a specific chemical component those spins will experience zero net excitation during excitation time on the resonance spins [8].

The magnetization transfer (MT) or magnetic transfer contrast (MTC) pulse technique is a spectrally selective RF pulse that reduces the MR signal from some types of tissues while leaving other types virtually unaffected [8].

The MT or MTC produces an RF pulse of width $\Delta f_{rf} > 0$ derived from a frequency offset that describes the RF envelope used, i.e., if a Gaussian or a Fermi pulse is used different SAR and flip angle will be present. Due to trade-offs on implementation the value of $\Delta f_{rf} \approx 1$ KHz.

On a different front of the spectrally selective pulses, atomic nuclei are surrounded by electrons which can shield the main magnetic field B_0 and reduce the net magnetic field experienced by nuclear spins [8]. Thus, due to the shielding protons in different microscopic environments can resonate at slightly different frequencies, known as *chemical shift*:

$$\delta = \frac{f - f_{TMS}}{f_{TMS}} \ \cdot\ 10^6 \quad \text{(parts per million} - \text{ppm)} \tag{6}$$

where f refers to the resonant frequency, f_{TMS} is the resonant frequency of tetramethyl silane. By convention, the zero *chemical shift* is assigned to the protons of in tetramethyl silane $Si(CH_3)_4$. Furthermore, the resonant frequency in relationship with the chemical shift yields:

$$f \ = \ \frac{\gamma}{2\pi}\ B_0\ (1-\delta) \tag{7}$$

where γ stands for the gyromagnetic ratio, and B_0 is the externally applied magnetic field.

B. Received (Rx) MRI Signal

The physical principles to detect the MR signal derived from Faraday's law electromagnetic induction are not different from a radio signal from a human-made transceiver for wireless communications to be detected with a radio receiver. Therefore, they are threaded in the general sense as two different waveforms that travel through free-space and are affected at least by additive white Gaussian noise. The two different waveforms present different natures; the receiver implementation might vary, which holds for any two distinct waveforms in wireless communications. As such, the physical principles of how an MR signal is generated, e.g., "*the principle of reciprocity*" are not discussed here, but they are thoroughly explained in [9]; instead, the frequency and time characteristics as waveform are of importance for the presented analysis.

The received signal y at the destination is viewed as a band-limited complex baseband signal that carries information within bandwidth $\mathscr{B}$ (see Fig. 1), defined as:

$$y = x * h + n \tag{8}$$

where x encapsulates the MR signal source, h the channel in the MR room, which can be modeled as a body wireless area channel, and n is the AWGN noise with power σ^2, and "*" the convolution operation. If proper equalization at the receiver is performed, the channel effects can be eliminated such that the arriving signal becomes:

$$y = x + n \tag{9}$$

The result expressed in equation (9) shows the received complex signal ready to be demodulated. Historically, the signal y has been related to a complex signal that contains phase and amplitude information [9]. However, if the transmitter and receiver are synchronized, the phase information is known. Consequently, the possible additional information found in the phase is related to channel impairments that can be removed with proper equalization methods.

In [9] the received demodulated complex signal is defined as:

$$\begin{aligned} y(t) &= \Re(y(t)) + \Im(y(t)) \\ &\approx \frac{1}{2}\cos(\delta\omega \cdot t - \theta) + \frac{1}{2}\sin(\delta\omega \cdot t - \theta) \\ &\approx \frac{1}{2}\left(\Re\left(e^{i\delta\omega \cdot t - i\theta}\right) + \Im\left(e^{i\delta\omega \cdot t - i\theta}\right)\right) \end{aligned} \quad (10)$$

where the real $\Re\{\cdot\}$ and imaginary $\Im\{\cdot\}$ are the parts of the complex acquired signal $x(t)$ that oscillates around the Larmor frequency, with the offset frequency $\delta\omega$, and phase θ. Now, if equations (10) and (9) are combined it yields:

$$\begin{aligned} y(t) &= \Re(y(t)) + \Im(y(t)) \\ &\approx \frac{1}{2}\cos(\delta\omega \cdot t - \theta) + \frac{1}{2}\sin(\delta\omega \cdot t - \theta) \\ &\approx \frac{1}{2}\left(\Re\left(e^{i\delta\omega \cdot t - i\theta}\right) + \Im\left(e^{i\delta\omega \cdot t - i\theta}\right)\right) + n(t) \end{aligned} \quad (11)$$

where the noise $n(t)$ is the AWGN noise with zero mean and variance σ^2, as an independent and identically distributed random variable (iid).

The signal $y(t)$ is already prepared for image reconstruction in classical MRI processing. In this work, the signal $y(t)$ is an amplitude modulated signal, e.g., as Pulse Code Modulation (PCM), that can be digitized to get the complex baseband signal $y[k]$.

The symbol synchronization between $x[k]$ and $y[k]$ can be made by finding the highest magnitude in the so-called k-space, where the highest magnitude is at the center.

a) Rx Signal on Coil Arrays as Single-Input Multiple-Output (SIMO) Wireless System: An extension of the model presented in Fig. 1 is the so-called Single-Input Multiple-Output wireless system presented in Fig. 3, where a coil array can process the arriving signal $y(t)$ for SNR improvement and spatial information, the Field-of-View (FoV). Spatial care of the sum of the received signals for all channels (coil receivers) should be considered for image generation; those methods are not treated here.

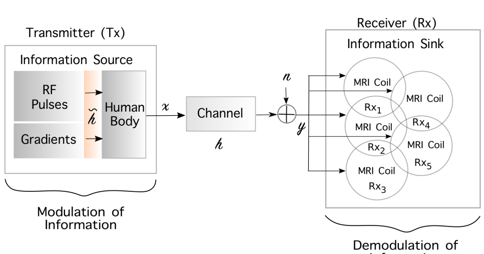

Fig. 3. Generalized wireless MRI Single-Input Multiple-Output (SIMO) communication model. The system can benefit from diversity and spatial gain to improve SNR

1) Molecule Impulse Response: RF pulses designed to selectively excite, refocus, or invert specific regions in the spectrum are called chemically (spectrally) selective pulses. In the frequency domain, one can estimate the NMR signal intensity incorporating the chemical shift illustrated by Eq. (6). Therefore, the instantaneous bandwidth can be estimated, Δf. The NMR impulse response in the frequency domain in the presence of a gradient in the frequency response is linearly related to the slice profile. Moreover, the instant bandwidth Δf is observed in the frequency domain in the absence of gradients during the RF pulse. The slice-selective excitation pulses are played concurrently with a slice selection gradient to produce an exited tissue section or slice-magnetization. Also, when hard-pulses with short duration are used, they excite all the magnetization coupled to the RF coil.

C. MRI Channel Model

For the sake of simplification, the adopted model to capture the channel statistics h inside the MRI room can be an indoor channel on a body area network [13].

III. Theoretical MRI Channel Capacity Estimation

As described in the introduction section, the parameters to calculate the theoretical channel capacity are the signal power, the channel bandwidth, and the present noise over the bandwidth and their relationship, i.e., the signal-to-noise ratio.

Several reference sources describe all parameters that constitute the calculation of the signal strength in the MRI environment. In this work the definition in equation (11) prevails. The definition in equation (11) includes the noise effects. The channel bandwidth is a variable resource as it depends on the MRI sequences used.

The SNR in MRI is considered as a function of the static magnetic field as appointed in [9], and it is relevant to distinguish one tissue from another.

A. MRI Signal Information Transmission Rate

The work described in [10] illustrates that from the ideal channel capacity definition in equation:

$$\begin{aligned} \hat{C} &= 2\,\mathscr{B}\,C \\ &= \mathscr{B}\,\log_2\left(1 + \frac{S}{N}\right) \\ &= \frac{\mathscr{B}}{\log_{10}(2)}\,\log_2\left(1 + \frac{S}{N}\right) \\ &\approx (3.3)\,\mathscr{B}\,\log_{10}\left(1 + \frac{S}{N}\right)\ \text{bits/s} \end{aligned} \quad (12)$$

for a bandlimited signal, the effective channel capacity is constrained by the sampling rate, quantization levels, and noise present yielding the expression:

$$C_{PCM} = \mathscr{B}\left(\log_2\left(3 + (n^2 - 1)k^2\right) - \log_2(3)\right) \quad (13)$$

where $n^{\xi} = Q$, and Q is the number of quantized levels. And $k = \frac{K}{\sigma}$ relates to the signal, where K is a constant and σ is the r.m.s. noise voltage. Applying the substitutions in equation (13), it yields:

$$C_{MRI} = \mathscr{B} \log_2 \left(1+k^2\right) \tag{14}$$

when $n=2$ describes a binary PCM MRI system that is sampled at $2\mathscr{B}$. The theoretical channel capacity of equation (14), and equation (12) are equivalent.

Recalling from equation of mutual information:

$$\begin{aligned} I(X;Y) &= H(X)-H(X|Y) = H(X)-(H(X,Y)-H(Y)) \\ &= H(Y)-H(Y|X) = H(Y)-(H(X,Y)-H(X)) \\ &= H(X)+H(Y)-H(X,Y) \\ &= I(Y;X) \end{aligned} \tag{15}$$

where $H(X)$ represents uncertanty about X before we know Y, $H(X|Y)$ representing the uncertainty after $H(X)-H(X|Y)$ is equal to the amount of information provided about X by Y. Mutuality needs to fulfill:

$$I(X;Y) \geq 0$$

the calculation of rate of transmission of information requires to know $H(x)$, $H(y)$ and $H(x,y)$. Taking the derivations from [10] the binary MRI PCM transmission rate follows as:

$$\begin{aligned} R_{MRI} = \mathscr{B} \Big[& \left(1+\operatorname{erf}\left(\frac{k}{\sqrt{2}}\right)\right) \log_2\left(1+\operatorname{erf}\left(\frac{k}{\sqrt{2}}\right)\right) + \\ & \left(1-\operatorname{erf}\left(\frac{k}{\sqrt{2}}\right)\right) \log_2\left(1-\operatorname{erf}\left(\frac{k}{\sqrt{2}}\right)\right)\Big] \end{aligned} \tag{16}$$

B. MRI Signal-to-Noise-Ratio

The MRI system's signal and noise ratio (SNR) has been studied mainly regarding the effects on the final image for diagnosis, including the signal to contrast ratio. However, the research in [14] provides insight into the SNR range of operation of a digital receiver and its dynamic range; in short, it covers the low and high SNR scenarios having the noise floor as a reference.

In this work, the noise is attributed to a random process that adds AWGN noise to the signal and using receiver methods, the received signal is improved. The noise model adds the thermal noise as well, which is considered a Gaussian random process, such that the random process of the AWGN stays without change.

Other aspects that reduce or improve the SNR include the quality of the coil receiver, including the low noise amplifier (LNA), the magnetic field strength of the MRI system under study, and the FoV.

The MRI wireless system considered in Fig. 1 assumes for the SNR calculation the maximum MR signal power level is at the echo moment, where all the spins are in phase, and there is no transversal magnetization left from previous acquisitions contributing to the signal of the current acquisition, i.e., no T2 memory. Last, the receiver's bandwidth determines the range of frequencies that can be received without degradation of signal quality beyond specified limits. The minimum required bandwidth is defined by the maximum acquisition (readout) gradient strength and the maximum diameter of the homogeneous region (FoV).

C. MRI Capacity Theorems

Theorem 1: The total channel capacity for a single echo acquisition and a single bandlimited coil receiver operating in the MRI system shall depend on the bandwidth of interest, the MR signal strength, and its relation to the noise n and channel h associated with it. *Proof:* Having the maximum of the mutual information over all possible statistics $\Pr\{X\}$:

$$\begin{aligned} C &= \sup_{\Pr\{X\}} \left(I(X;Y)\right) \\ &= \sup_{\Pr\{X\}} \Big(\sum_{\nu}\sum_{\mu} \Pr\left\{Y_\mu|X_\nu\right\} \Pr\left\{X_\nu\right\} \\ &\log_2 \left(\frac{\Pr\left\{Y_\mu|X_\nu\right\}}{\sum_l \Pr\left\{Y_\mu|X_l\right\} \Pr\left\{X_l\right\}}\right)\Big) \quad \text{bits/s/Hz} \end{aligned} \tag{17}$$

where the sup is the "supremum" operant, the largest value of $I(X;Y)$ as it varies. If the MRI channel capacity is derived from equation (17), a suitable alphabet $\mathscr{A}$ shall exist for its symbol detection, as demonstrated in equation (14). ■

Theorem 2: The hard pulses generate more information compared to the slice selective pulses. *Proof:* If a channel capacity equation is derived for the hard-pulses $C_{Hard-Pulses}$, the slice selective pulses capacity $C_{Slice-Selective}$ must be $C_{Slice-Selective} \leq C_{Hard-Pulses}$ since they consume less bandwidth following the behavior described in equation (14). ■

Theorem 3: The total channel capacity of an MRI sequence shall be the average of the single-channel capacity overall acquired echoes. *Proof:* If the bandwidth of interest $\mathscr{B}$ of a sequence acquisition changes over time during its execution, the channel capacity and the transmission rate will change following the description of equations (14) and (16) respectively. ■

IV. Results

The scenario where the MRI signal acquisition is sampled at the receiver follows the constraints:

- All voltages in the MRI as PCM system both transmitted and received will be referred to unit resistance.
- The PCM amplitude modulation scheme is performed at the receiver for demodulation.
- The MRI system provides synchronization between transmitter and receiver.
- The aggregated noise is AWGN, in which the thermal noise is included.
- Equalization is performed to reduce the effects of fading in the channel.
- Assuming the maximum MR signal power level is at the echo moment, where all the spins are in phase.
- Assuming no transversal magnetization, left from previous acquisitions contributing to the signal of the current acquisition, e.g., no T2 memory.

- A single-coil element for the reception of the signal, i.e., a SISO model for the MRI wireless communication, is assumed, as shown in Fig. 1.
- At the receiver, the signal is sampled at the speed of $2\mathscr{B}$.

Fig. 4 demonstrates the total efficiency of the MRI wireless communication system when the signal is assumed to be amplitude modulated. It is relevant to highlight that increasing the bandwidth $\mathscr{B}$ will have a positive impact on its efficiency to a certain point. However, by improving the SNR (K/σ) the total efficiency will be reduced. This effect is due to the theoretical channel capacity increases while the data transfer rate settles as described by the equation (16).

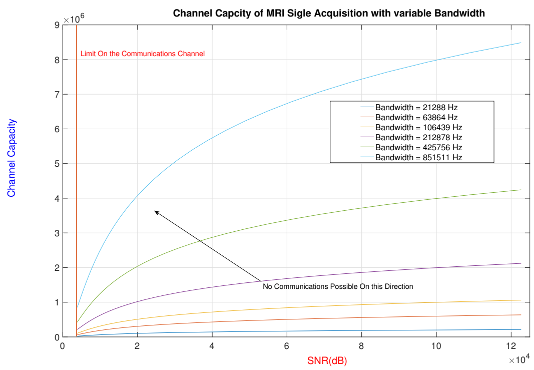

Fig. 4. Data Transfer Efficiency with several ranges of bandwidth $\mathscr{B}$ as the MRI signal is treated as a PCM signal, for a single Echo acquisition.

To further explain the data transfer rate and the channel capacity relationship, see Fig. 5 where the maximum data transfer rate of the PCM MRI system:

- follows the theoretical channel capacity with a loss offset,
- after certain SNR value (towards high SNR) the data transfer rate of the system settles with no further improvement,
- the increase in bandwidth helps to increase the data transfer rate with a ceiling limit.

Moreover, suppose the typical MRI receiver bandwidth ranges from one KiloHertz to one MegaHertz. In that case, the performance of the system follows the same behavior as shown in Fig. 6.

The receiver demodulates the signal as an amplitude modulated signal sent from the transmitter. However, it would have the same performance as if the signal was modulated in phase and amplitude as stated in [11]. Moreover, one might argue that there is still some information to be captured in the received signal phase. However, changes in phase translate to changes in frequency. In other words, if the bandwidth of interest is defined a priori, there is no change in the overall system performance.

To the end, if the receiver is prepared to demodulate the received signal in phase and frequency, the performance can be calculated as described in [11], i.e., having a transmitter modulating the signal in amplitude or in-phase and demodulating the signal in amplitude or phase or a combination of both.

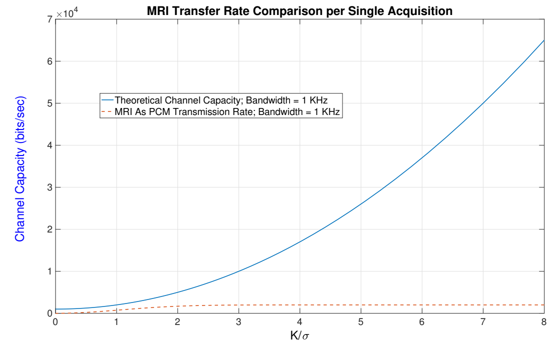

Fig. 5. MRI transfer rate comparison per single acquisition, with $\mathscr{B}$= 1 KHz, as the MRI signal is treated as a PCM signal.

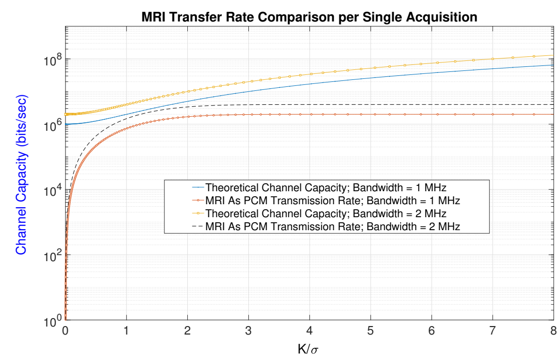

Fig. 6. MRI transfer rate comparison per single acquisition, with $\mathscr{B}$= 1 MHz, and $\mathscr{B}$= 2 MHz, as the MRI signal is treated as a PCM signal.

V. Discussion

The calculation of the channel capacity and transmission rate of a communication system brings the question of how to optimize the communication link in more than one way. In the MRI system, the calculation of the maximum limits at which the information can flow from the human body to the coil array shall describe the fastest sequence for the image acquisition. For instance, research in the area of image acquisition acceleration is taking place, Spiral and EPI sequences are fast methods to cover the k-space, compressed SENSE reduces the number of samples required to reconstruct an image, the combination of linear and non-linear gradients suggest higher encoding efficiency[15]. However, before this work, there was no theoretical maximum limit rate in which the system coupled with the sequence can receive the data to generate the final image. Furthermore, the modeling of the MRI system as a communication system as depicted in Fig. 3 suggests that each voxel provides a piece of information spatially encoded as part of a massive MIMO array, spatial MIMO modulation. Therefore, the question in the MRI system resembles a question in the communications systems: which modulation technique could perform as close as possible to the maximum theoretical channel capacity and transmission rate? The quest to answer this question in the

MRI system should be an open research path to explore from information theory as exposed by this work. This work does not include the algorithms and architecture implementations to achieve information capacity due to the model structure. Those are left to the engineers who design the MRI system from including the aspects of coil receivers to the image reconstruction for diagnosis.

VI. Conclusions

In this work, the fundamentals concepts of information theory are applied to the MRI system to set a functioning structure to achieve information capacity, that is, receiving information efficiently. To this end, the MRI system is presented as a wireless communication system. The MRI signal is treated as an amplitude modulated signal where the information about the body is embedded into it. Therefore the amount of information can be calculated. The consequences of calculating the system capacity illustrate the theoretical limits to capture the received data at the coil with absolute confidence of not discarding valuable information, knowing the maximum data compression is the data entropy rate. The results point out the importance of knowing the top performance values of the system under ideal circumstances and the inherent signal degradation due to noise and the limits imposed by the MRI signal waveform. The MRI waveform at its heart is an inefficient waveform to transport information as its frequency harmonic contents are limited.

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