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Mathematical Analysis of Population of Bursting Neurons

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***Abstract*—Rhythmic activity of the neural population with bursting neurons is important in brain information processing and some diseases, but the role of bursting neurons remains unknown. Thus, in this study, we proposed a new model of bursting neuron and clarified how the characteristics of bursting neurons affect the generated rhythms of neural population.**

***Clinical Relevance*— Bursting is thought to be important in some neuropathologies such as epilepsy. Therefore, revealing the dynamics of population of bursting neurons may lead to the elucidation of the mechanism of epilepsy.**

I. INTRODUCTION

The synchronized activity of bursting neurons, a type of neuron in the cerebral cortex that fires the cluster of spikes, is thought to play a role in brain information processing, diseases such as epilepsy [1], and the control of the rhythmic activity of neuron populations. However, theoretical analysis of the dynamics of a population of neurons including bursting neuron is difficult due to the high dimensionality of bursting neuron models, and its dynamics is still unclear. To clarify the effect of bursting neuron on the collective rhythm, we proposed a simple and biologically parameterized bursting neuron, and performed bifurcation analysis and derivation of the phase sensitivity curve for the population model.

II. METHODS

To model bursting neuron simply, we added a term that changes slowly depending on the membrane potential $I_{\mathrm{slow}} = -\cos(\theta/n)$ to the bias current I of the modified theta (MT) model supposed by Kotani et al. [2]. This bursting neuron model is described as

$$C\frac{d\theta}{dt} = -g_{\mathrm{L}}\cos\theta + \frac{2}{V_{\mathrm{T}} - V_{\mathrm{R}}}(1+\cos\theta)(I + I_{\mathrm{slow}}), \qquad (1)$$

where the membrane capacitance $C = 1$ (μF cm^{-2}), firing threshold $V_{\mathrm{T}} = -55$ (mV), resting potential $V_{\mathrm{R}} = -62$ (mV), θ is the phase corresponding to the membrane potential, n in I_{slow} is the number of spikes induced by a single burst. Then, we consider a neural population consisting of 400 excitatory bursting neuron models and 100 inhibitory MT models with chemical synapses. With the mean-field approximation for synapses, we derived the Fokker-Planck equation describing the dynamics of each neural population and the ordinary differential equation describing the behavior of the mean-field synapses. Bifurcation analysis was performed on these models to identify the parameters that result in limit cycle solutions. Then, the phase response curve (PRC) for the limit cycle solution was derived from both adjoint method and perturbation method.

III. RESULTS

Fig. 1 shows the time course of membrane potential $V = (V_{\mathrm{T}} + V_{\mathrm{R}})/2 + (V_{\mathrm{T}} - V_{\mathrm{R}})/2\cdot\tan(\theta/2)$ of the proposed model described in (1) for each n. When n is 2 or more, Fig. 1 shows the periodic bursting pattern.

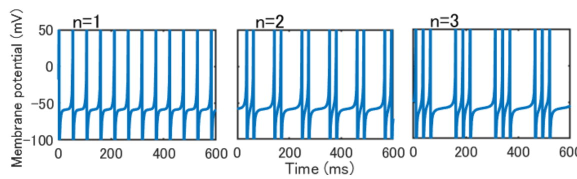

Figure 1. The time course of membrane potential of bursting neuron.

Fig. 2 (A) shows the bifurcation of the population model for n = 1 and n = 2. The population model oscillates in the area above the bifurcation line. The bifurcation line of n = 2 shifts to the left side compared to n = 1. Fig. 2 (B) shows the PRC for the rhythmic activity of excitatory population by adjoint method (line) and perturbation method (dot).

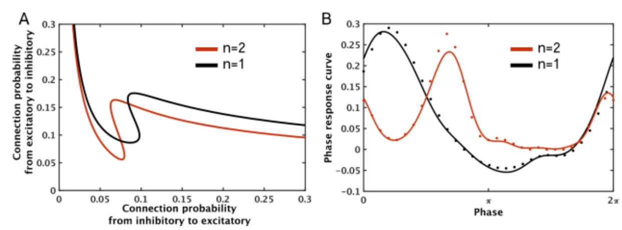

Figure 2. Bifurcation line and PRC of the neural population

IV. DISCUSSION & CONCLUSION

The shift of the bifurcation line in Fig. 2 (A) means an increase in the strength of the coupling from the excitatory to the inhibitory population. The shapes of the PRC mean the population with bursting neurons can be synchronized to faster external rhythms. These findings suggest a further role of bursting neurons in the rhythmic activities.

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