An Efficiency Based Approach for the Energy Management in HEVs

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Abstract: A proper Energy Management Strategy (EMS) is a cornerstone for Hybrid Electric Vehicles consumption minimization. In general, to find the global optimal strategy, the knowledge of the entire mission profile of the vehicle is needed, making the real-time implementation impossible. A well-known solution to this problem is the Equivalent Consumption Minimization Strategy (ECMS) that optimizes at each time instant an equivalent fuel consumption, which combines the real fuel rate and a virtual fuel associated to the use of the battery energy. This virtual fuel is the actual battery power weighted by an equivalence factor, that implicitly accounts for the battery recharge efficiency during the vehicle mission. In this work, we propose an efficiency based EMS, rather than a fuel consumption based one. Despite the two approaches are proven to be identical under some assumptions, in the efficiency based solution the definition of the equivalence factor results easier. An offline estimation of this quantity is firstly proposed, eventually extended with a real-time adaptation. Simulation results show the effectiveness of the proposed approach, in particular when the adaptive strategy is used.

Keywords: Hybrid vehicles, Energy management, Efficiency optimization, Fuel saving, ECMS

1. INTRODUCTION

Hybridization is an intermediate (possibly long lasting) step in the road towards the global vehicle electrification. Up to now, most of the hybrid vehicles belong to the Hybrid Electric Vehicles (HEVs) class, which merges the high energy and power density of fossil-based fuels – employed by the internal combustion engines (ICE) – with the high efficiency of electric motors (EM), fueled by the energy stored in the onboard battery pack [Onori et al. (2016)]. The presence of multiple power sources requires the development of strategies to govern the various energy flows, called Energy Management Strategies (EMSs). Their aim is the overall vehicle efficiency optimization, in order to reduce consumption, emissions, pollution and also costs.

Different EMSs have been developed and widespread in literature. Among them, the Equivalent Consumption Minimization Strategy (ECMS) has a relevant place [Paganelli et al. (2001)]; in fact, this solution is very effective [Serrao et al. (2009)] and simple to be implemented through pre-computed offline maps [Sivertsson and Eriksson (2015)]. In order to be used in real-time, ECMS solves an optimization problem at any time instant minimizing the equivalent consumption, that is the sum of the fuel consumption and the battery power weighted by an equivalence factor, which takes into account how the battery is recharged during the use of the vehicle. Despite the optimization occurs at any time instant, it has been shown that this solution is very close to the global optimal one, computed when the entire mission profile is a-priori known [Serrao et al. (2011)]. Moreover, it is also proven that under some assumptions [Kim et al. (2012)], a possibly time varying equivalence factor exists, such that the solution coincides with the optimal one [Serrao et al. (2009)]. Therefore, many works have been oriented to the

definition of an adaptive strategy that updates the equivalence factor in real-time [Onori et al. (2010); Yang et al. (2018, 2021)] or at least its bounds [Rezaei et al. (2018)].

In this work, we propose an alternative energy management strategy based on the explicit instantaneous maximization of the vehicle efficiency. Similarly to ECMS, the knowledge of an equivalent efficiency at which the battery is recharged is needed. Nevertheless, differently from the ECMS approach, the required equivalence factor can be more easily estimated, thanks to the physical interpretation of the proposed optimization problem. Firstly, we point out the steps for an offline estimation of its value. Then, we develop an adaptive estimation strategy based on past driving information. Additionally, we also show how the proposed Efficiency Maximization Problem (EMP) can be set within the ECMS framework, taking advantage of its well-validated properties and features.

The efficiency maximization strategy is tested in simulation environment on a parallel HEV, representing an urban car equipped with a 86 kW engine mounted in parallel with a 30 kW electric motor before a manual transmission. Simulations reveal how fuel saving is close to the optimal solution, when the equivalent efficiency is computed using its physically inspired definition. In view of avoiding the a-priori knowledge of the vehicle mission profile, the effectiveness of the adaptive solution is eventually shown.

The reminder of the paper is organized as follows: in Section 2, the problem is defined along with the vehicle model used for the simulation campaign. In Section 3, the background on ECMS is recalled before the presentation of the efficiency based EMS in Section 4, which is finally validated in Section 5.

2. PROBLEM DEFINITION AND SIMULATION ENVIRONMENT

The considered case study aims at optimizing the fuel consumption of a non plug-in urban parallel HEV. The vehicle powertrain is characterized by a 86 kW thermal engine in parallel with a 30 kW electric motor, coupled with the wheels through a manual transmission $¹$. The vehicle behavior is reproduced in</sup> Matlab/Simulink² environment, simulating the vehicle energy consumption associated to the tracking of a longitudinal speed profile. The model can be summed up in the following set of equations:

$$
\begin{cases}\n\dot{v} = \frac{1}{M} \left(\frac{T_{ice} + T_{em}}{R_w \tau_{gb} \tau_0} - F_{cd} \right) \\
\dot{m}_f = \frac{P_f}{\lambda_f} \\
\text{SoC} = -\frac{P_b}{Q_b}\n\end{cases}
$$
\n(1)

The first equation, where *M* is the total vehicle mass, is the result of the longitudinal force balance, where the engine (*Tice*) and motor torque (T_{em}) – scaled at the wheels by the gear box ratio τ_{gb} , the final drive one τ_0 and the wheel radius R_w – are visible along with the coasting-down force *Fcd*, needed to drive the vehicle at constant speed *v*:

$$
F_{cd} = Cv^2 + Bv + A\cos(\theta) + Mg\sin(\theta). \tag{2}
$$

The coasting-down force includes the aerodynamic (Cv^2) and viscous (*Bv*) friction, the rolling resistance *A* influenced by the vehicle slope θ and finally the gravitational effect Mg , which is visible when the slope is different from zero.

The last two equations in (1) are required to compute the energy consumption rates, expressed in terms of fuel mass m_f and battery state of charge SoC. Fuel consumption is derived scaling the fuel power P_f by fuel power density λ_f , and SoC by dividing the battery power P_b with the battery energy capacity Q_b . Fuel and battery power are linked with the mechanical power provided to the vehicle through their respective efficiency maps:

$$
P_f = \frac{T_{ice}\Omega}{\eta_{ice}} \quad \text{and} \quad P_b = \frac{T_{em}\Omega}{\eta_{em}^{\text{sign}T_{em}}}.
$$
 (3)

 η_{ice} is the engine efficiency and η_{em} includes the motor and inverter one, shown in Fig. 1 and 2, respectively. In these equations, the rotational speed Ω appears, that is the same both for ICE and EM because of the mechanical nature of the considered parallel HEV. The complete model parameters are reported in Tab. 1.

3. EQUIVALENT CONSUMPTION MINIMIZATION STRATEGY BACKGROUND

Before presenting the proposed energy management strategy, the traditional background of ECMS is briefly recalled. The aim of EMS in a HEV is the consumption minimization over its entire life, by optimally using the different power sources. When considering a charge-sustaining scenario, where the battery state of charge is self-maintained by the engine, the battery acts as an energy buffer and so the consumption to be minimized is associated to the thermal engine fuel only. Given that the optimal solution of this problem can be achieved only when the mission profile of the vehicle is a-priori known [Sciarretta

Fig. 1. Engine efficiency map.

Fig. 2. Electric motor and inverter efficiency map. Torque limits are different in traction and recharge to be compatible with power battery constraints $(\bar{P}_b^d, \bar{P}_b^r)$.

Table 1. Vehicle parameters used in simulation environment.

param.	value	unit
M	1200	kg
R_{W}	28.1	cm
τ_0	3.17	
τ_{gb}	(3.42, 1.96, 1.28, 0.94, 0.76)	
\overline{A}	93.1	N
B	2.54	N/(m/s)
$\mathcal{C}_{0}^{(n)}$	0.38	$N/(m/s)^2$
	9.81	$(m/s)^2$
$\overset{g}{\lambda}_f$	43.308	MJ/kg
	4.5	kWh
	30	kW
$Q_b \bar{P}_b^d \bar{P}_b^r$	15	kW

et al. (2004)], a possible solution is the Equivalent Consumption Minimization Strategy (ECMS) [Paganelli et al. (2001)] that turns this problem into a feasible one for real-time applications. In fact, it is reduced to a local optimization at any time instant of an equivalent consumption *weq*:

$$
\min\left\{w_{eq}(t) = \frac{P_{eq}(t)}{\lambda_f}\right\}, \forall t,
$$
\n(4)

where *Peq* is the equivalent power consumption:

$$
P_{eq} = P_f + \lambda P_b. \tag{5}
$$

It must be recalled that while the fuel power is always positive, the electrical power of the battery could assume also negative values; this happens when the battery is recharging. The parameter λ appearing in equation (5) is the so-called *equivalence factor* that makes the two power sources comparable, in terms

¹ Vehicle and engine parameters available online in the "Advanced Light-Duty" Powertrain and Hybrid Analysis (ALPHA) Tool", see also Lee et al. (2013).

² Mathworks, Portola Valley, California, USA.

of energy consumption. Indeed, the efficiency at which the battery is recharged must be taken into account. In particular, when the battery state of charge is self-maintained by the thermal engine itself, each battery consumption is associated to a past or future fuel consumption to recharge the battery. Therefore, all the information on the complete efficiency chain encountered to recharge the battery should be included in the definition of λ [Onori et al. (2016)]. Moreover, it has been demonstrated by Serrao et al. (2009) – thanks to the Pontryagin's Minimum Principle (PMP) – that, despite the problem simplification, under some assumptions [Kim et al. (2012)], ECMS solution is equivalent to the optimal global one for a particular and possibly time varying value of $\lambda = \lambda^*(t)$. Given that the knowledge of the vehicle profile is needed to find $\lambda^*(t)$, it cannot be computed in real-time, therefore different strategies have been developed to at least provide a prior estimate for its bounds [Rezaei et al. (2018)] or a real-time estimation [Onori et al. (2010)] using various inputs, such as the driving style [Yang et al. (2018)] or the past driving information [Yang et al. (2021)].

Given these premises and considering the specific case of parallel HEVs in exam, the optimization variable becomes the sole torque split α between the engine and motor ones

$$
\alpha: T_{ice} = \alpha T_{ref} \text{ and } T_{em} = (1 - \alpha) T_{ref}. \tag{6}
$$

Notice that the previous definition easily enforces the constraint related to the driver power request $P_{ref} = T_{ref} \Omega$, that must be always matched. Therefore, the ECMS is the solution of the following optimization problem:

$$
\min_{\alpha} \left[P_f(\alpha, \Omega, T_{ref}, \eta_{ice}) + \lambda P_b(\alpha, \Omega, T_{ref}, \eta_{em}) \right] \forall \Omega, T_{ref}.
$$
 (7)

Exploiting the expression of P_f and P_b and recalling that in parallel HEVs engine and electric motor rotational speed are mechanically coupled, the cost function in (7) can be rewritten as:

$$
\min_{\alpha} \left[\frac{\alpha T_{ref}}{\eta_{ice}(\Omega, \alpha T_{ref})} + \lambda \frac{(1-\alpha)T_{ref}}{\eta_{em}(\Omega, (1-\alpha)T_{ref})} \right] \forall \Omega, T_{ref}.
$$
 (8)

The main information for the solution of the optimization problem are thus the engine and motor efficiency maps. The optimal solution can be pre-computed off-line, stored as a map [Sivertsson and Eriksson (2015)] and then retrieved online by using current measure of engine speed and torque request. Due to the reduction of the EMS to a local optimization at any time instant, any terminal charge-sustaining constraint cannot be explicitly taken into account. To cope with this issue, many works propose an indirect solution [Kleimaier and Schroder (2002); Kessels et al. (2008); Chasse et al. (2010)], by manipulating the physical meaning of λ in order to weight the battery power in function of the SoC, making the battery use more or less convenient. In fact, the use of the battery power can be forced to be more convenient at high SoC, decreasing λ , and vice-versa.

In (8) the physical interpretation of the parameter λ – as source of information on the efficiency chain encountered to recharge the battery – is visible, as it acts as a scaling factor on the electric motor efficiency. Despite this intuition, the estimation of the proper value of the equivalence factor is not a simple task. For example, Rezaei et al. (2018) suggest using the ratio of average motor and engine efficiency while Yang et al. (2021) propose an adaptive technique monitoring the vehicle behaviour during its use. In the next section, we will tackle this problem by formulating the EMS in terms of efficiency maximization, providing a physical expression that can be used to more easily determinate the equivalence factor value.

4. EFFICIENCY MAXIMIZATION PROBLEM

The proposed EMS is based on the definition of efficiency for a hybrid vehicle, focusing in particular on parallel HEVs. Similary to ECMS, the efficiency is maximized at any time instant, making the solution easily implementable as pre-computed maps.

4.1 Efficiency definitions

Generally speaking, efficiency is defined as the ratio between the produced power and the consumed one [Rizzoni et al. (1999)]; due to the reversibility of electric machines, the efficiency assumes two different expressions, considering if the electric motor is used as a motor, applying positive torques, or generator, when the torque is negative. Looking at the block diagram in Fig. 3, the power contributions in vehicle usage are the fuel power consumption P_f , the battery power P_b and the power requested P_{ref} to counteract the vehicle load P_l .

Fig. 3. Power flow scheme for efficiency definition in HEVs.

When the engine is used not only to provide power to the load but also to charge the battery $(P_b < 0 -$ called *recharge mode*), the efficiency η^r assumes the following definition:

$$
\eta^r = \frac{P_{ref} - P_b}{P_f}, P_b < 0. \tag{9}
$$

On the other hand, when battery and fuel are both used by the load, discharging the battery $(P_b \ge 0 - \text{called } \text{discharge mode}),$ the efficiency $\eta^{\bar{d}}$ has a slightly different expression:

$$
\eta^d = \frac{P_{ref}}{P_f + P_b}, P_b \ge 0.
$$
\n(10)

Considering that, in particular for non plug-in vehicles, the battery energy does not come for free, but it is supplied by the ICE or by the regenerative braking, a more suitable definition for the *discharge mode* efficiency includes the *equivalence efficiency* $\tilde{\eta}^r$. It accounts for the information of the entire vehicle life efficiency chain during recharge phases (11), playing the same role of $λ$ in ECMS:

$$
\eta^d = \frac{P_{ref}}{P_f + \frac{P_b}{\overline{\eta}^r}}, P_b \ge 0.
$$
\n(11)

However, $\tilde{\eta}^r$ can be precisely computed, thanks to definition (9), rewritten in an energy form:

$$
\tilde{\eta}^r = \frac{E_{ref} - E_b}{E_f} = \frac{\int_{\mathcal{F}_r} P_{ref} dt - \int_{\mathcal{F}_r} P_b dt}{\int_{\mathcal{F}_r} P_f dt},\tag{12}
$$

where \mathcal{T}_r coincides with the time intervals in recharge:

$$
\mathcal{F}_r = \{ t \in [0, +\infty) : P_b(t) < 0 \}. \tag{13}
$$

Recalling the fact that in non plug-in vehicles the battery can be recharged by the engine or by regenerative braking, the recharge efficiency is written as follows:

$$
\tilde{\eta}^r = \frac{\int_{\mathcal{T}_{r,ice}} P_{ref} dt - \int_{\mathcal{T}_{r,ice}} P_b dt - \int_{\mathcal{T}_{r,rb}} P_b dt}{\int_{\mathcal{T}_{r,ice}} P_f dt},\tag{14}
$$

where:

$$
\mathcal{F}_{\text{r,ice}} = \left\{ t \in [0, +\infty) : P_b(t) < 0 \land P_f(t) > 0 \right\},\n\mathcal{F}_{\text{r,rb}} = \left\{ t \in [0, +\infty) : P_b(t) < 0 \land P_f(t) = 0 \right\},\n\tag{15}
$$

showing that the energy coming from regenerative braking is not associated to any fuel consumption, increasing the efficiency in recharge [Kleimaier and Schroder (2002)].

4.2 Efficiency Maximization Problem

The introduced efficiencies are here used to formulate an energy management problem for a parallel HEV. In particular, the proposed EMS computes the optimal torque split α , defined in (6), so to maximize the total efficiency:

$$
\max_{\alpha} \eta = \begin{cases} \eta^r(\alpha, \Omega, T_{ref}, \eta_{ice}, \eta_{em}) & T_{em} < 0\\ \eta^d(\alpha, \Omega, T_{ref}, \eta_{ice}, \eta_{em}, \tilde{\eta}^r) & T_{em} \ge 0 \end{cases} \forall \Omega, T_{ref}.
$$
\n(16)

The solution of the problem, named EMP (Efficiency Maximization Problem) requires the knowledge of the efficiency value $\tilde{\eta}^r$, that according to (12) is available only a-posteriori, sharing the same challenge of the equivalence factor λ estimation in ECMS (7). Nevertheless, the physical expression provided in (12) can be used to develop two different solutions, a prior offline estimate $\bar{\eta}^r$ and a real-time adaptation $\bar{\eta}^r(t)$.

Offline estimation. The offline procedure to estimate $\tilde{\eta}^r$ is characterized by the following steps:

(1) find the optimal torque split during recharge in each operating point $(Ω, T_{ref})$:

$$
\alpha^{r,opt}(\Omega, T_{ref}) = \arg \max \eta^r(\alpha, \Omega, T_{ref}); \qquad (17)
$$

(2) compute the associated optimal efficiency map:

$$
\eta^{r,opt} = \eta^r(\alpha^{r,opt}(\Omega, T_{ref}), \Omega, T_{ref}); \tag{18}
$$

(3) compute the constant estimation $\bar{\eta}^r$ of the recharge efficiency as the average value of $\eta^{r, opt}$ over the speed-torque plane:

$$
\tilde{\eta}^r \approx \bar{\eta}^r.
$$

Despite the procedure is performed offline, this estimation makes a step forward than using just the average value of engine and motor efficiency, as recalled by Rezaei et al. (2018). In fact, the average value $\bar{\eta}^r$ includes also the information coming from the maximization of the efficiency η^r . Nevertheless, the (typical) limitation of the offline estimation is the lack of information on how the driving-cycle and driving-style impact the equivalent efficiency in recharge. Moreover, the equivalent efficiency increase due to regenerative braking, as shown in (12), is not taken into account. Therefore, the offline estimator can be used to provide an initial value for the online procedure, presented in the following. Finally, it should be noticed that, once found $\bar{\eta}^r$, the solution of (16) can be pre-computed into maps function of the current operating point.

Online adaptation. The proposed online adaptation strategy is based on the computation of the equivalent efficiency using the definition in (12) and (13) and the information collected up to the current time instant:

$$
\bar{\eta}^r(t) = \frac{\int_{\mathcal{F}_r(t)} P_{ref} dt - \int_{\mathcal{F}_r(t)} P_b dt}{\int_{\mathcal{F}_r(t)} P_f dt},\tag{19}
$$

where

$$
\mathscr{T}_r(t) = \{ \tau \in [0, t) : P_b(\tau) < 0 \}. \tag{20}
$$

The adaptive estimation can be initialized at the value $\bar{\eta}^r$ provided by the offline estimation (4.2), turning (19) into:

$$
\bar{\eta}^r(t) = \frac{E_0 + \int_{\mathcal{T}_r(t)} P_{ref} dt - \int_{\mathcal{T}_r(t)} P_b dt}{\frac{E_0}{\bar{\eta}^r} + \int_{\mathcal{T}_r(t)} P_f dt},\tag{21}
$$

where E_0 is a weight on the initial value with respect to the data collected in real-time: increasing E_0 the trust in the initial value increases, and vice-versa. The resulting adaptive scheme is shown in Fig. 4. As one can see, the real-time estimation of the recharge equivalent efficiency is based on standard available measures: P_{ref} is known form driver request, P_b by multiplying battery voltage and current and P_f by multiplying the fuel rate and λ_f .

Fig. 4. Adaptive scheme for equivalent efficiency in recharge estimation.

In conclusion, in the efficiency based optimization problem (16) the recharge efficiency is replaced by its adaptive estimate:

$$
\tilde{\eta}^r \approx \bar{\eta}^r(t).
$$

Notice that, when the adaptive estimate is used, at each time instant a new optimization problem must be solved and the offline pre-computation of its solution cannot be longer employed. However, the static nature of the optimization problem does not raise any particular concern on its real-time feasibility.

4.3 Relationship with ECMS

The proposed EMP (16) is here proven to be equal to ECMS (7) for a suitable choice of λ . The first step consists in writing the ECMS into a maximization problem:

$$
\arg\min P_f + \lambda P_b = \arg\max \frac{P_{ref}}{P_f + \lambda P_b}, \forall \Omega, T_{ref}.
$$
 (22)

Therefore, it is sufficient to find a value of λ such that the following equivalence holds:

$$
\arg\max \frac{P_{ref}}{P_f + \lambda P_b} = \arg\max \left\{ \frac{\eta^r}{\eta^d} \frac{P_b < 0}{P_b \ge 0}, \forall \Omega, T_{ref}.\tag{23}
$$

• Recalling (11), the equivalence with ECMS in discharge phases is immediately visible, setting λ as the inverse of the equivalent efficiency in recharge:

$$
\frac{P_{ref}}{P_f + \lambda P_b} = \eta^d = \frac{P_{ref}}{P_f + \frac{P_b}{\overline{\eta}^r}} \to \lambda = \frac{1}{\overline{\eta}^r}.
$$
 (24)

• Recalling efficiency expression in (9), the equivalence in (23) in recharge phases, holds when:

$$
\frac{P_{ref}}{P_f + \lambda P_b} = \eta^r = \frac{P_{ref} - P_b}{P_f} \tag{25}
$$

and, as consequence, when λ assumes the following value:

$$
\lambda = \frac{P_f}{P_{ref} - P_b} = \frac{1}{\eta^r}.\tag{26}
$$

Fig. 5. Optimal solutions for different constant values $\hat{\eta}^r$. In red areas the electric motor provides a positive torque, while a negative one in the blue ones. Between the engine maximum torque (gray) and the total torque (black) the electric motor torque must always provide a positive torque to satisfy the reference requested by the driver.

Finally, efficiency optimization and ECMS are equivalent if:

$$
\lambda = \lambda(t) = \begin{cases} \frac{1}{\eta^r(t)} & P_b(t) < 0 \\ \frac{1}{\tilde{\eta}^r} & P_b(t) \ge 0 \end{cases} \tag{27}
$$

The proof of the equivalence between EMP with a particular realization of ECMS makes the proposed efficiency based approach belonging to a well validated framework for the energy management in hybrid vehicles. For example, the chargesustaining constraint, not considered up to now, can be easily introduced as in ECMS, simply by adding an additional tunable weight γ on the battery power, as a function of the SoC:

$$
\max \eta_{eq} = \begin{cases} \frac{P_{ref} - \gamma P_b}{P_f}, & P_b < 0\\ \frac{P_{ref}}{P_f + \gamma \frac{P_b}{\hat{\eta}^r}}, & P_b \ge 0 \end{cases} \forall \Omega, T_{ref}.
$$
 (28)

5. SIMULATION RESULTS

The EMP is tested on a case study based on the parallel HEV presented in Section 2. The validation is carried out by neglecting the charge-sustaining constraint in order to focus on the optimality of the proposed solution. First of all, the solution of the problem in (16) is computed for a set of constant parameters $\hat{\eta}^r$. Some examples of the resulting optimal torque maps are shown in Fig. 5, including also the one where the offline estimated recharge efficiency $\bar{\eta}^r = 0.33$ is used.

These different optimal strategies are tested on two different driving-cycles (highway and urban), represented in Fig. 6.

Fig. 6. Driving-cycles used for simulations: highway (ArtMW130) and urban (ArtRoad).

The aim of this first analysis is the computation of the optimal constant value $\hat{\eta}^{r, opt}$ on the specific profile, which allows to evaluate the optimality performance of the constant offline estimator $\bar{\eta}^r$. Given the different final SoC for each value of $\hat{\eta}^r$, the equivalent fuel saving, see e.g. Sciarretta et al. (2004) or Yang et al. (2021), is used to compare the different strategies; to this purpose the equivalence factor is computed a-posteriori with (12) , on the charge-sustaining solution.

Fig. 7. Equivalent fuel saving performance in the highway and urban driving-cycle.

The percentage of equivalent fuel saved with respect to the ICEonly consumption, for different values of $\hat{\eta}^r$, is shown in Fig 7. The following considerations can be drawn:

- (1) the constant offline estimator value ($\hat{\eta}^r = \bar{\eta}^r = 0.33$), is associated to a significant fuel saving, but not the optimal one;
- (2) the optimal constant value (estimated using the fitted line) depends on the specific profile, resulting $\hat{\eta}^{\tau, opt} = 0.398$ for the highway and $\hat{\eta}^{r, opt} = 0.412$ for the urban cycle;
- (3) the equivalent recharge efficiency computed a-posteriori using (12) ($\tilde{\eta}^r = 0.399$ for highway and $\tilde{\eta}^r = 0.410$ for urban) is very close to the optimal constant value $\hat{\eta}^{r, opt}$.

The last point suggests how the adaptive solution presented in Section 4 could improve the overall fuel saving, without any prior knowledge on the considered profile, and only leveraging past driving data. The estimation is initialized with the offline estimate $\bar{\eta}^r$ and tends towards the optimal constant value $\hat{\eta}^{r, op}$ computed a-posteriori in Fig 7. The performance of the adaptive strategy is quantified in Tab. 2, comparing its equivalent fuel saving with the one experienced applying the constant offline estimator $\bar{\eta}^r$ and the optimal constant one $\hat{\eta}^{r, opt}$. The results show that adaptive solution performance is higher than using the offline estimator and is close to the performance experienced with $\hat{\eta}^{r, opt}$. Finally, in Fig. 8, the optimality of the solution is evaluated comparing the results of the adaptive EMP with the global optimal solution that reaches the same final

Table 2. Equivalent fuel saving performance comparison.

	offline EMP		adaptive EMP		optimal const. EMP	
	\bar{n}'	eq. fuel	$\bar{\eta}^r(t_{end})$	eq. fuel	$\hat{n}^{r,opt}$	eq. fuel
highway	0.33	4.44%	0.393	5.96%	0.398	6.07%
urban	0.33	20.19%	0.413	24.92%	0.412	27.70%

SoC. Due to the HEV model assumptions in (1), the optimal solution can be computed, as shown in Serrao et al. (2009) and Kim et al. (2012), by iteratively founding the constant costate that satisfies PMP conditions constraining the final SoC to be equal to the one reached by the adaptive EMP. Results – summarized in Tab. 3 – reveal the effectiveness of the proposed energy management strategy, which is able to achieve real fuel saving performance close to the optimal global solution.

Fig. 8. SoC and *Tem* comparison between PMP and adaptive EMP with $\bar{\eta}^r(t)$ (highway and urban profile).

Table 3. Real fuel saving performance in L/100km of the adaptive EMP and PMP solution compared with the ICE-only configuration.

L/100km $L/100km$ (%) $4.87(-6.5%)$ highway 5.22	PMP	adaptive EMP	ICE-only	
	$L/100km$ (%)			
	4.84 (-7.1%)			
	$2.91(-25.2\%)$	$2.95(-23.9\%)$	3.88	urban

6. CONCLUSIONS

In this paper, an efficiency maximization based energy management strategy for HEVs is presented. This approach is equivalent to the classic ECMS one for a proper choice of the equivalence factor, but has the advantage of providing a more intuitive way to estimate its value. Simulations show that an offline calibration is able to provide good results, when compared with the ones of an optimal constant value found exploiting the apriori knowledge of the driving cycle. Moreover, the proposed adaptive solution improves the fuel saving, getting close to the optimal global solution. In this simulation campaign, the performance is evaluated in terms of optimality without including the necessity of guaranteeing charge-sustaining that, leveraging the equivalence with the EMCS strategy, could be accounted for in a similar way.

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